Problem 1.

Compute the following limits if they exist. Show enough work to justify your computation, or your claim that the limit does not exist.

(a)

$$\lim_{x \to -2} \frac{x^2 + 6x + 8}{2(x+4)(x+2)} =$$

(b)

$$\lim_{x \to 9} \frac{3 - \sqrt{x}}{9 - x}$$

(c)

$$\lim_{x \to -\infty} \frac{3x^3 + \sqrt[3]{x}}{\sqrt{9x^6 + 2x^2 + 1} + x}$$

(d)

$$\lim_{x \to 1^+} \frac{|x - 1|}{x - 1} =$$

Problem 2.

Compute the following limits if they exist. Show enough work to justify your computation, or your claim that the limit does not exist.

(a)

$$\lim_{x\to 1} \frac{\sin^2(x-1)}{(x-1)^2} =$$

(b)

$$\lim_{x\to -2} \frac{x^2+6x+9}{2(x+4)(x+2)} =$$

(c)

$$\lim_{x\to \pi} \frac{\sin(x)}{x} =$$

(d)

$$\lim_{x\to 3} \frac{x-5}{(x-3)^2} =$$

Problem 3. (a) Using the Squeeze Theorem, show that

$$\lim_{x \to 3} \frac{x-3}{1+\sin^2\left(\frac{2\pi+e+7}{x-3}\right)} = 0.$$

(b) Let

$$g(x) = \begin{cases} \frac{x^2-1}{x-1} & x > 0 \\ x^2+1 & x < 0 \end{cases}$$

If possible, define an extension of g that is continuous at all real numbers.

Problem 4. (a) Show that the polynomial $x^4 - 6x - 2$ has two real roots, that is, there are two (different!) real numbers a and b such that $a^4 - 6a - 2 = b^4 - 6b - 2 = 0$.

(b) Directly from the definition of derivative, compute $f'(x)$ if $f(x) = \sqrt{x+3}$.

Problem 5. Compute the following derivatives using only the definition of derivative.

(a) Derivative of $f(x) = x^2 + \sqrt{x}$ at $x = 2$.

(b) Derivative of $g(x) = \frac{1}{x+1}$ at $x = 1$.