Math 114 Practice Exam 3

Problem 1. You may use any methods we have learned in class to solve these problems, but show enough work to justify your answers.

(a) Find $\frac{d^2f}{dx^2}$ if $f(x) = x\cos x$.

(b) If $g(x) = \sin(3x)$ compute $g'(\pi/12)$

(c) Find an equation of the line tangent to $y = \frac{x^2-1}{x^2+1}$ at the point $(0, -1)$.

Problem 2. Compute the derivatives of the following functions using methods we have learned in class. Show enough work to justify your answers.

(a) $f(x) = \sec\left(\frac{\sqrt{x^2+1}}{x+2}\right)$

(b) $g(x) = \sqrt[4]{\frac{x^3+\cos(x^2)}{\sin(x^3)+1}}$

Problem 3. (a) Find a formula for y' in terms of x and y if $x^8 + x^4 + y^4 + y^6 = 1$.

(b) Find a tangent line to the curve given by $x^4 - 2x^2y^2 + y^4 = 16$ at the point $(\sqrt{5}, 1)$.

(c) If $x^2y = x + y$ find a formula for $\frac{d^2y}{dx^2}$ in terms of x and y.

Problem 4. (a) It is a fact that $2^{10} = 1024$. Estimate 2.01^{10} using the derivative of x^{10} at the point 2.

(b) Use a linear approximation to estimate $\sqrt{4.01}$.

(c) Suppose we have the differential equation $f'(t) = f(t) - t$, with $f(1) = 2$. Use Euler's method with three steps to approximate $f(4)$.

Problem 5. (a) The surface area of a cube is given by the formula $A = 6s^2$ where s is the length of a side. If the side lengths are increasing by 2 inches per second, how fast is the surface area increasing when the area is 54 square inches?

(b) A car is driving down a road at 150 feet per second (this is about a hundred miles an hour). A camera is placed 200 feet from the road, which will rotate to follow and record the progress of the car. How quickly must the camera rotate when the car is fifty feet away from directly in front of the camera?