

$$\int_R f dA = \int_a^b \int_c^d f(x, y) dy dx$$

Volume of region bounded by

$$z = x + y, z = 10, x = 0, y = 0 \Rightarrow x + y \leq 10 \quad y \leq 10 - x$$

$$\int_0^{10} \int_0^{10-x} (10 - (x+y)) dy dx = \int_0^{10} \left(10y - xy - \frac{y^2}{2} \Big|_0^{10-x} \right) dx$$

$$= \int_0^{10} 100 - 10x - 10x + x^2 - \frac{1}{2}(100 - 20x + x^2) dx$$

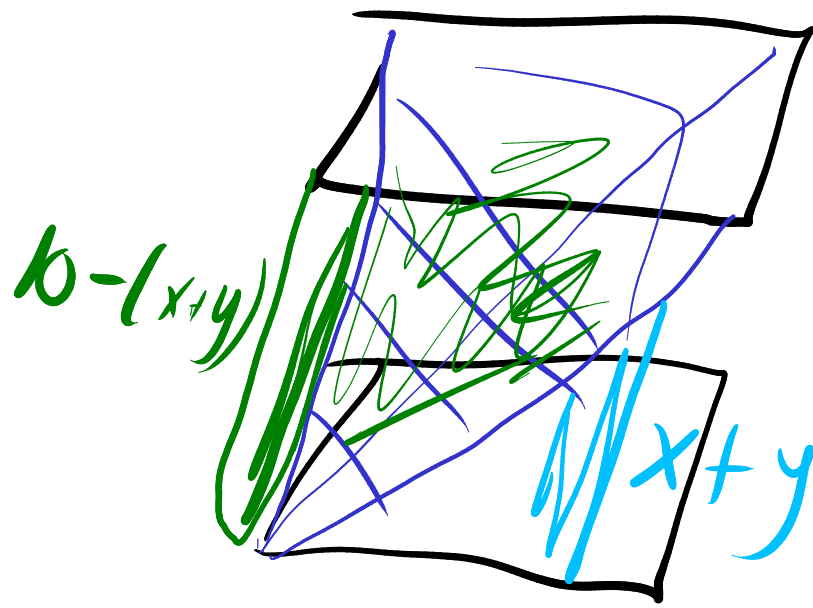
$$= \int_0^{10} 50 - 10x + \frac{x^2}{2} dx = 500/6.$$

Volume of region bounded by

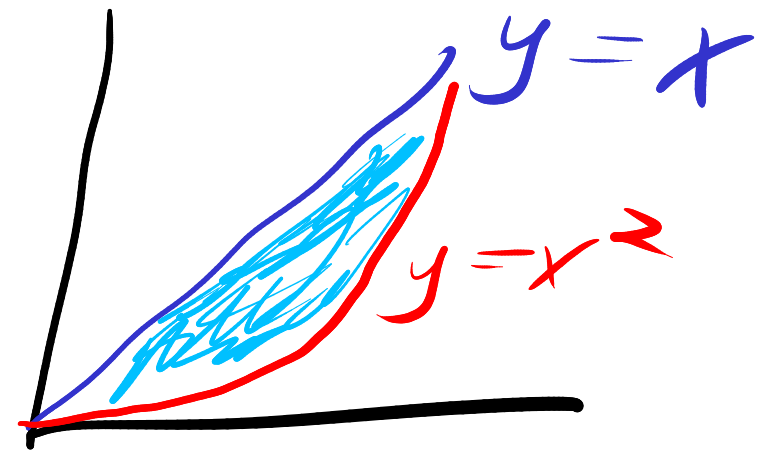
$$z = x + y, z = 10, x = 0, y = 0 \Rightarrow x + y \leq 10 \quad y \leq 10 - x$$

$$\int_0^{10} \int_0^{10-x} \int_{x+y}^{10} 1 \, dz \, dy \, dx =$$

$$= \int_0^{10} \int_0^{10-x} 10 - (x+y) \, dy \, dx$$



Average value of $f(x, y) = y(x-1)$
on R bdd by $y=x, y=x^2$



$$\int_0^1 \int_{x^2}^x y(x-1) dy dx = \int_0^1 \frac{y^2}{2} (x-1) \Big|_{x^2}^x dx = \dots = -\frac{1}{40}$$

$$\int_0^1 \int_y^{\sqrt{y}} y(x-1) dx dy = \dots = -\frac{1}{40}.$$

is the total value
of f on R

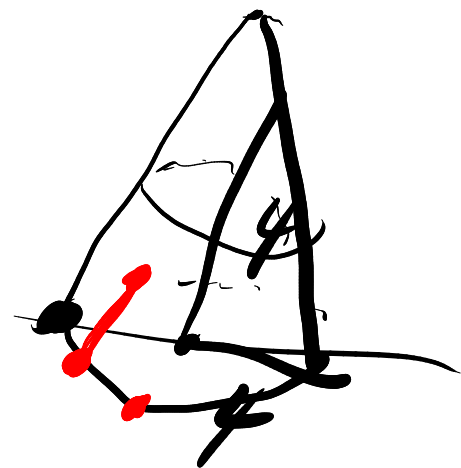
$$A = \frac{1}{6}$$
$$\text{Average} = \frac{-\frac{1}{40}}{\frac{1}{6}} = -\frac{3}{20}$$

Mass of cone bdd by xy plane

$$\text{and } z = 4 - \sqrt{x^2 + y^2} \quad \sqrt{x^2 + y^2} \leq 4$$

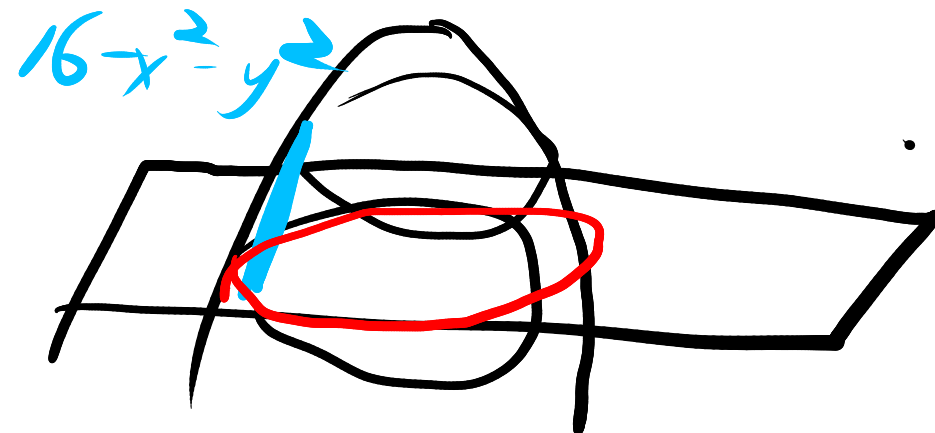
$$\text{density is } \delta(x, y, z) = xz \quad x^2 + y^2 \leq 16$$

$$\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_0^{4-\sqrt{x^2+y^2}} xz \, dz \, dy \, dx$$



Volume of solid below $f(x,y) = 25 - x^2 - y^2$

above $z = 9$



$$\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} (16 - x^2 - y^2) dy dx$$

$$\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_9^{25-x^2-y^2} 1 dz dy dx$$

5.15

5.20

~~4~~

Integrate x^2y over upper half of unit circle

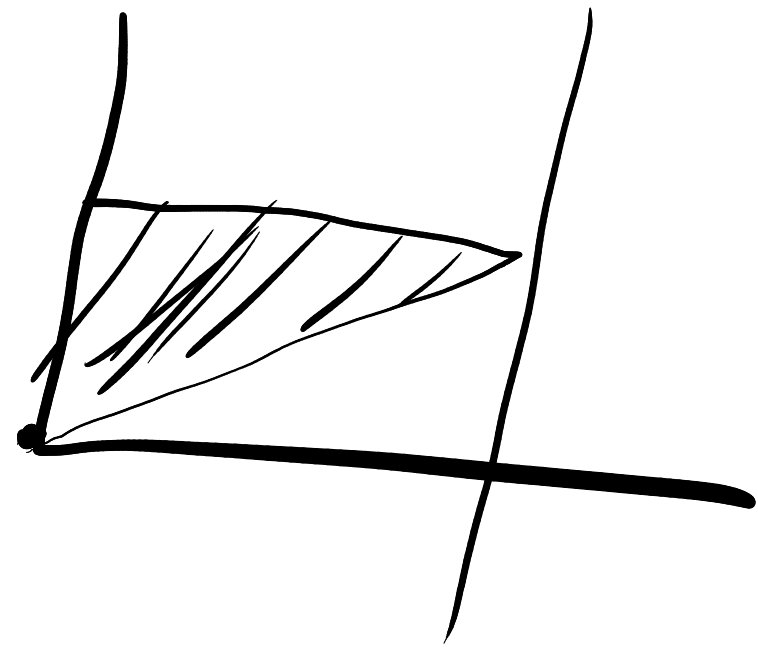
$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} x^2 y \, dy \, dx = \int_{-1}^1 x^2 \frac{y^2}{2} \Big|_0^{\sqrt{1-x^2}} \, dx$$
$$= \int_{-1}^1 x^2 \frac{1-x^2}{2} \, dx = \frac{2}{15}$$

integrate $x^2 y^2$ over D

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} x^2 y^2 dy dx = \int_{-1}^1 x^2 \frac{y^3}{3} \Big|_0^{\sqrt{1-x^2}} dx$$

$$= \frac{1}{3} \int_{-1}^1 x^2 (1-x^2)^{3/2} dx = \dots = \frac{1}{144} \left(x\sqrt{1-x^2} (-8x^4 + 14x^2 - 3) + 3 \arcsin(x) \right) \Big|_{-1}^1$$

$$5.17 \quad \int_0^6 \int_{x/3}^{\sqrt{3}x} x \sqrt{y^3+1} \, dy \, dx$$



$$= \int_0^2 \int_0^{3y} x \sqrt{y^3+1} \, dx \, dy = \int_0^2 \frac{x^2}{2} \sqrt{y^3+1} \Big|_0^{3y} \, dy$$

$$= \int_0^2 \frac{9y^2}{2} \sqrt{y^3+1} \, dy = \frac{(y^3+1)^{3/2}}{3} \Big|_0^2 = 27 - 1 = 26.$$