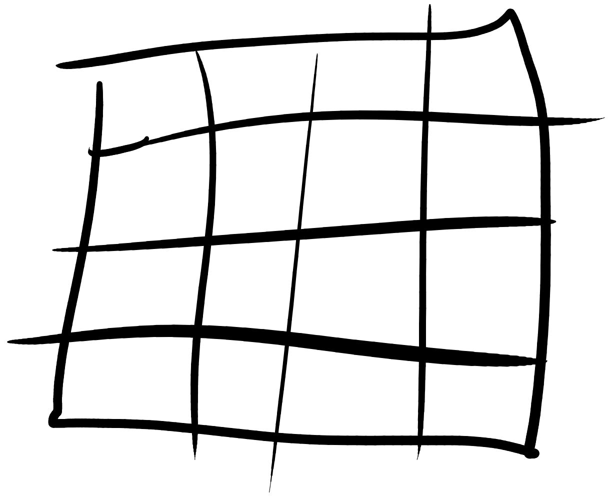
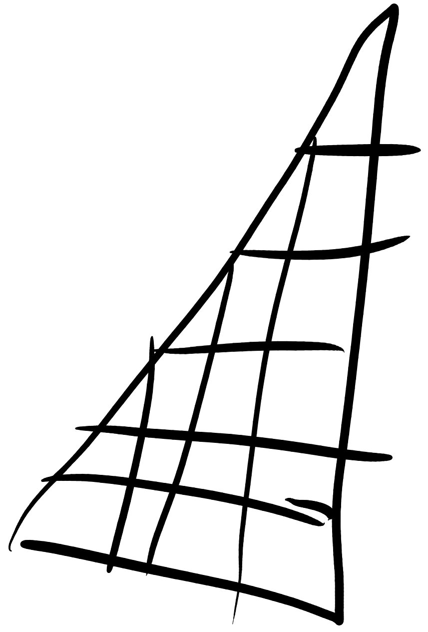
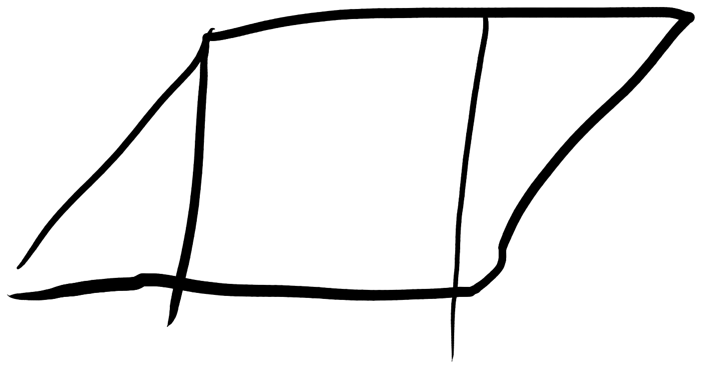
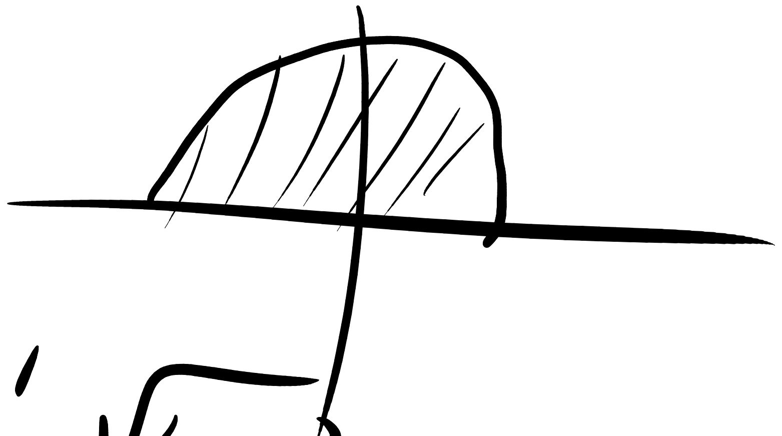
§5.3 Polar coordinates

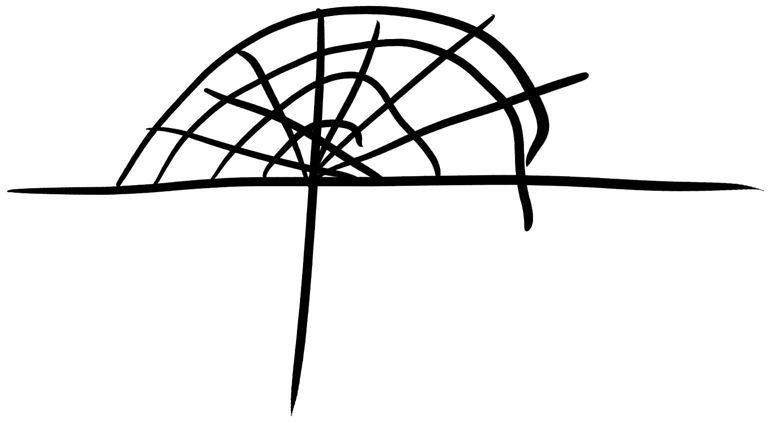




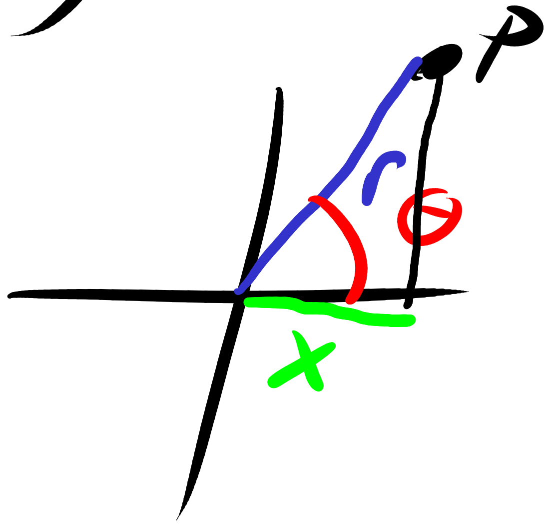




$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}$$



Dfn: polar coordinates pair (r, θ)



r: distance btwn P and O

θ: $\angle$ btwn $\vec{i}$ and $\overrightarrow{OP}$

Prop: $P = (x, y) = (r, \theta)$

$r \geq 0$

$0 \leq \theta < 2\pi$

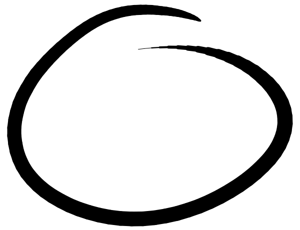
$x = r\cos\theta \quad y = r\sin\theta$

$r = \sqrt{x^2+y^2} \quad \theta = \pm \arctan\frac{y}{x}$

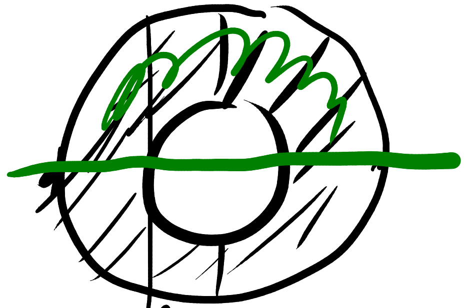
eqn for circle

$x^2+y^2=c^2$

$r=c$

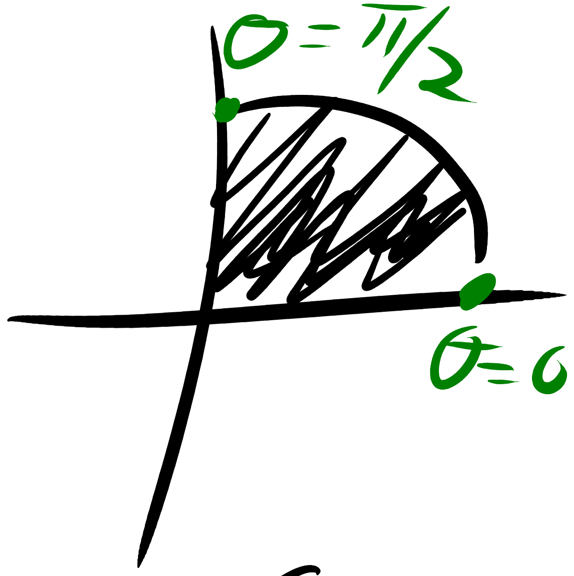


$\{1\le r\le 2,\ 0\le\theta\le 2\pi\}$



washer
annulus

$\{1\le r\le 2,\ 0\le\theta\le\pi\}$



Closed unit disk
upper right quarter

$\{(r,\theta)\,|\,0\le r\le 1,\ 0\le\theta\le\pi/2\}$

$\{(x,y)\,|\,0\le x\le 1,\ 0\le y\le\sqrt{1-x^2}\}$

$$\int_0^1\int_0^{\sqrt{1-x^2}} dy\,dx \quad\Bigg|\quad \int_0^1\int_0^{\pi/2} d\theta\,dr$$

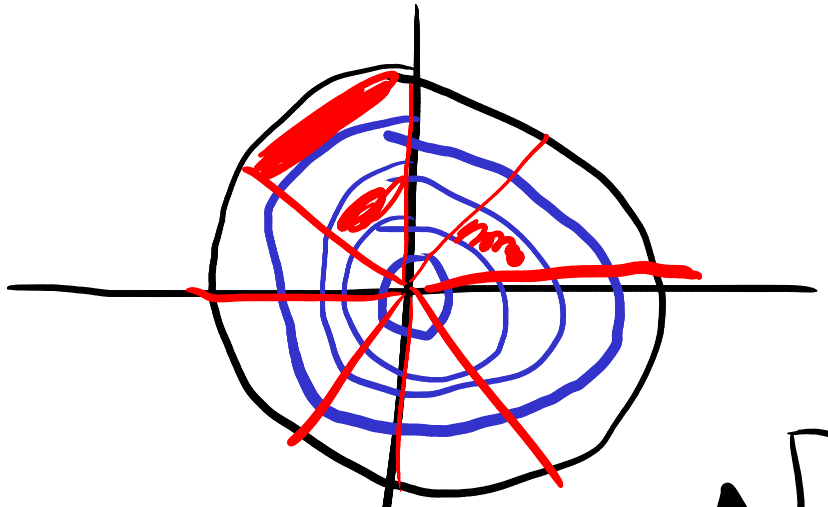
$$x = 3 \Rightarrow r\cos\theta = 3$$

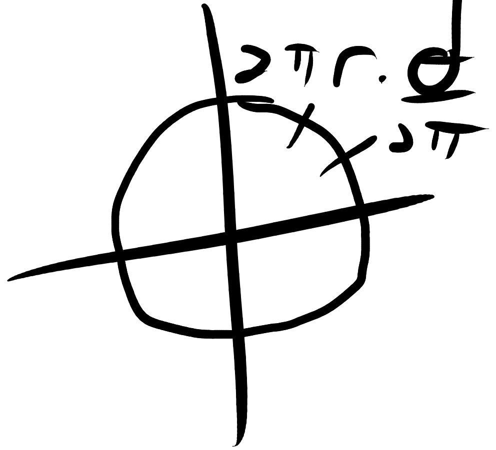
$$\cos\theta = 3/r$$

$$\sum f(x^*, y^*)\,\Delta x\,\Delta y$$

$$\sum f(r^*, \theta^*)\,\Delta r \cdot \Delta\theta \cdot r$$

$$= \sum f \cdot r\;\Delta r\,\Delta\theta$$



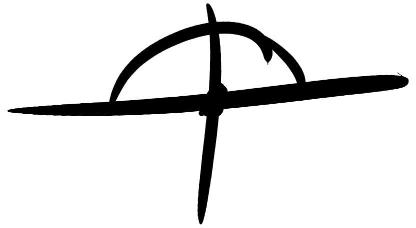


$$I = \int_{r_1}^{r_2} \int_{\theta_1}^{\theta_2} f(r,\theta) \cdot r \, d\theta\, dr$$

inner radius $= r$

outer radius $= r + dr$

in the limit, $r \approx r + dr$

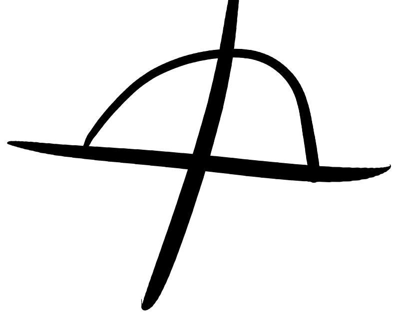
integrate x^2y over 

$$I = \int_0^1 \int_0^\pi \underbrace{r^2\cos^2\theta}_{x^2}\,\underbrace{r\sin\theta}_{y}\,\underbrace{r}_{r}\,d\theta\,dr$$

$$= \int_0^1 \int_0^\pi r^4\cos^2\theta\sin\theta\,d\theta\,dr = \int_0^1 r^4\,\frac{-1}{3}\cos^3\theta\Big|_0^\pi\,dr$$

$$= \int_0^1 \frac{2}{3}r^4\,dr = 2/15$$

x^2y^2 over upper unit semicircle

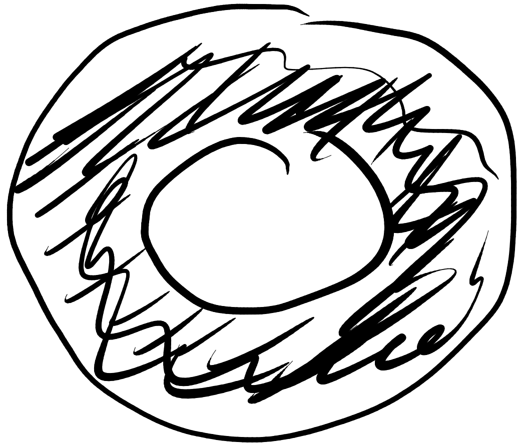


$$\int_0^1\int_0^\pi r^2\cos^2\theta\cdot r^2\sin^2\theta\cdot r\,d\theta\,dr$$

$$=\int_0^1\int_0^\pi r^5\cos^2\theta\sin^2\theta\,d\theta\,dr=\int_0^1 r^5\left(\frac{\theta}{8}-\frac{1}{32}\sin(4\theta)\right)\Big|_0^\pi dr$$

$$=\int_0^1 r^5\frac{\pi}{8}\,dr=\frac{r^6\pi}{48}\Big|_0^1\to\pi/48.$$

integrate $\dfrac{1}{\sqrt{x^2+y^2}}$ over a annulus inner radius 1 outer radius 2



$$\int_0^{2\pi}\int_1^{2} \frac{1}{r} \cdot r \, dr \, d\theta = \int_0^{2\pi}\int_1^{2} 1 \, dr \, d\theta = 2\pi.$$

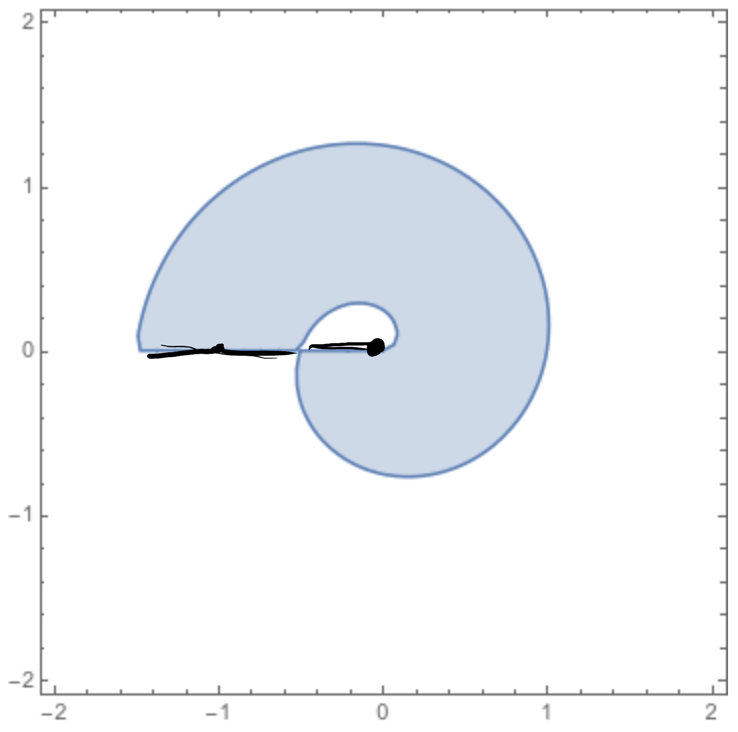
$$\int_1^{2}\int_0^{2\pi} \frac{1}{r} r \, d\theta \, dr$$

$$\frac{1}{\sqrt{r^2\sin^2\theta + r^2\cos^2\theta}} = \frac{1}{r\sqrt{\sin^2+\cos^2}} = \frac{1}{r}$$

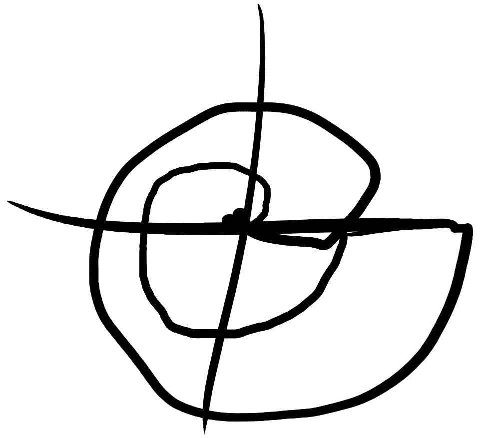
Find the area of spiral thickness 1,
inner radius goes from 0 to 1

$$\int_0^{2\pi}\int_{\frac{\theta}{2\pi}}^{1+\frac{\theta}{2\pi}} 1\cdot r\,dr\,d\theta$$

"line" through
(0,0), (2π, 1)
(0,1) (2π, 2)



$$=\int_0^{2\pi} \frac{r^2}{2}\Bigg|_{\frac{\theta}{2\pi}}^{\left(1+\frac{\theta}{2\pi}\right)} = \frac{1}{2}\int_0^{2\pi} 1+\frac{\theta}{\pi}\,d\theta = 2\pi$$



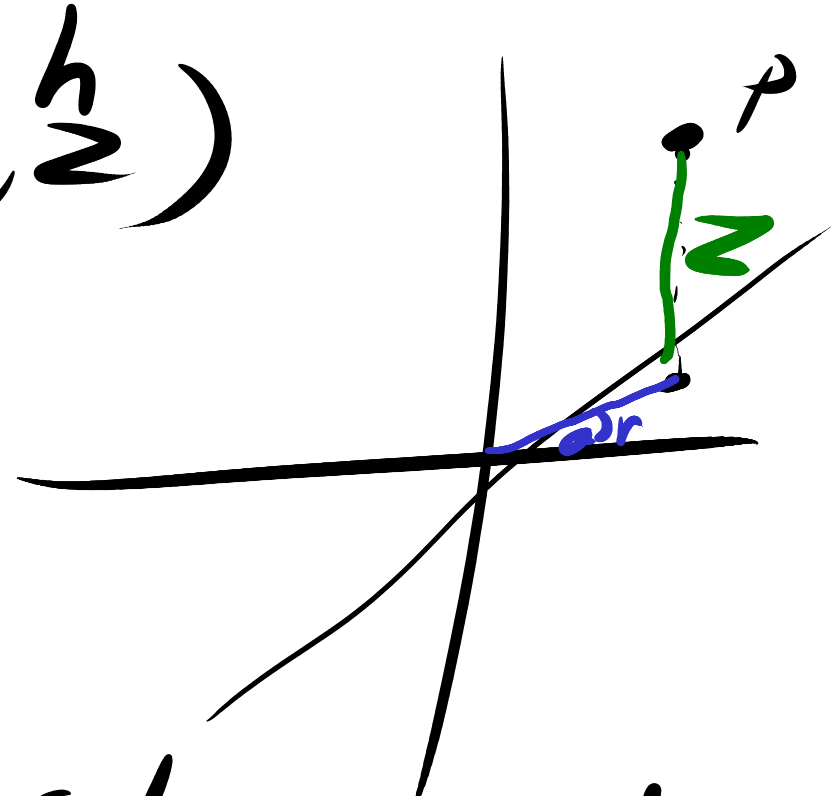
$$\int_0^{2\pi}\int dr\,d\theta$$

§5.4 Cylindrical and Spherical coords

Dfn: Cylindrical coords $(r, \theta, \overset{h}{z})$

turn x, y into polar coords

keep z for the height.



$x = r\cos\theta$ $\quad y = r\sin\theta$ $\quad z = h$

$r = \sqrt{x^2+y^2}$ $\quad \theta = \pm\arctan y/x$ $\quad h = z$

Cylinder around z-axis radius 3

$x^2+y^2=9$ $\qquad r=3$