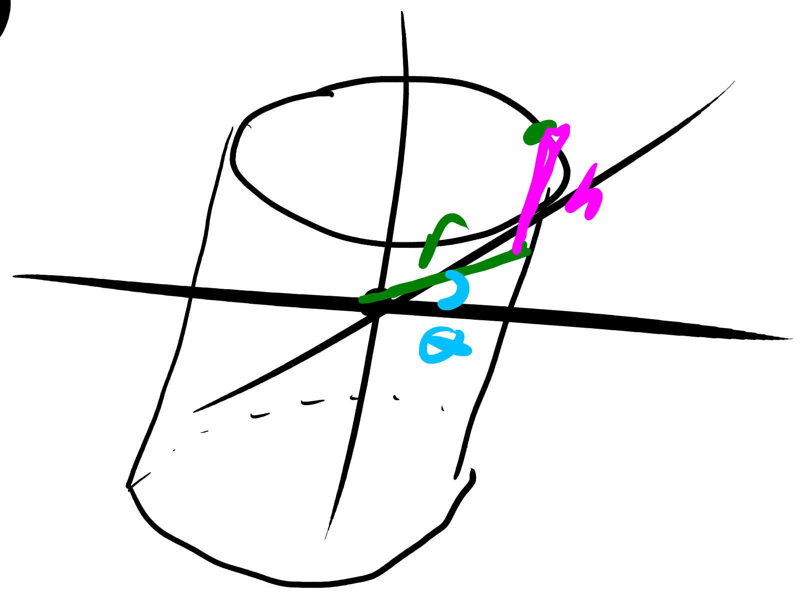
§5.4 Cylindrical and Spherical Coordinates

Cylindrical: $(r, \theta, h) \mid r \geq 0,\ \theta \in [0, 2\pi)$

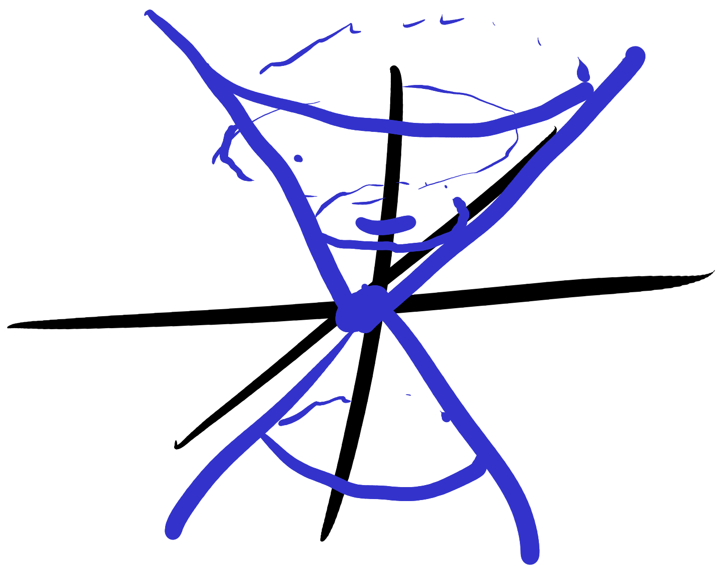


$x = r\cos\theta \qquad y = r\sin\theta \qquad z = h$

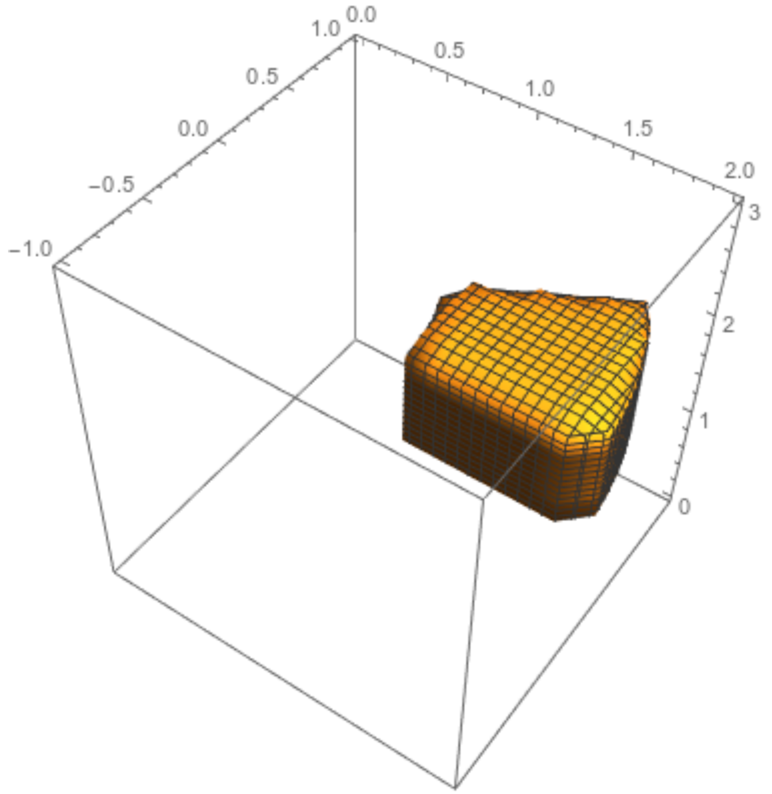
$r = \sqrt{x^2+y^2} \qquad \theta = \pm\arctan y/x \qquad h = z$

$r = c$ cylinder

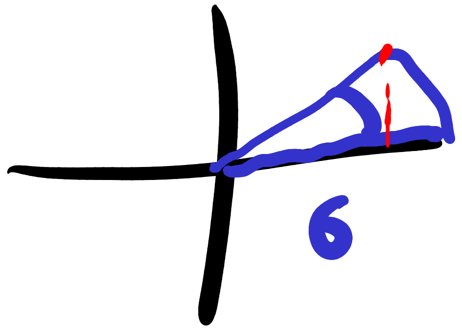
$r = h$ cone



$$dV = r\,dr\,d\theta\,dh$$



Integrate xz over wedge cut from a cylinder 4 high, 6 radius angle $\pi/6$.

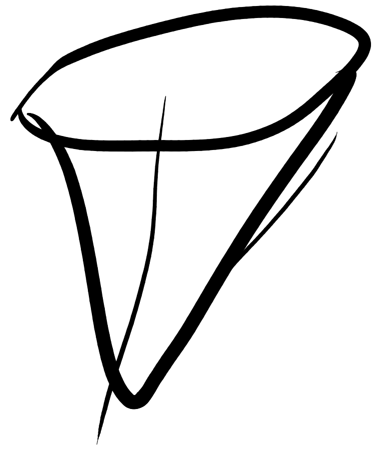


$$\int_0^4 \int_0^6 \int_0^{\pi/6} r\cos\theta\, h \; r \, d\theta\, dr\, dh$$

$$= 288$$

Integrate xyz over cone

$0 \le z \le 4, \quad x^2+y^2=z^2$



$$\cancel{\int_0^4 \int_{-z}^{z} \int_{-\sqrt{z^2-x^2}}^{\sqrt{z^2-x^2}} xyz \, dx \, dz}$$

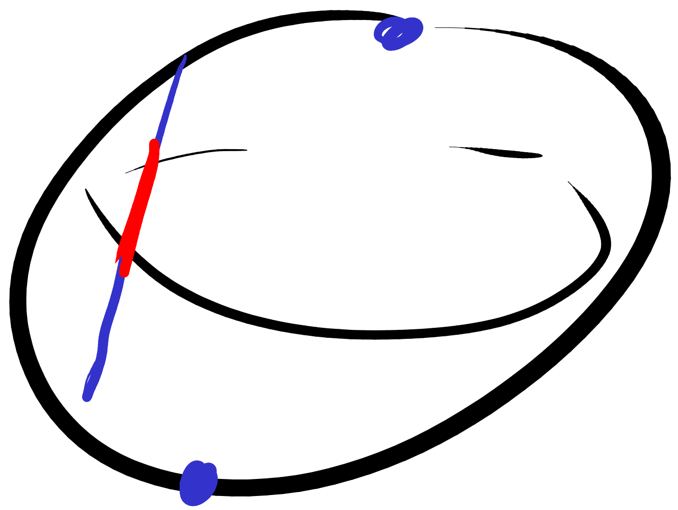
$r^2 = h^2$

$$\int_0^4 \int_0^h \int_0^{2\pi} r^2 \cos\theta \sin\theta \, h \, r \, d\theta \, dr \, dh =$$

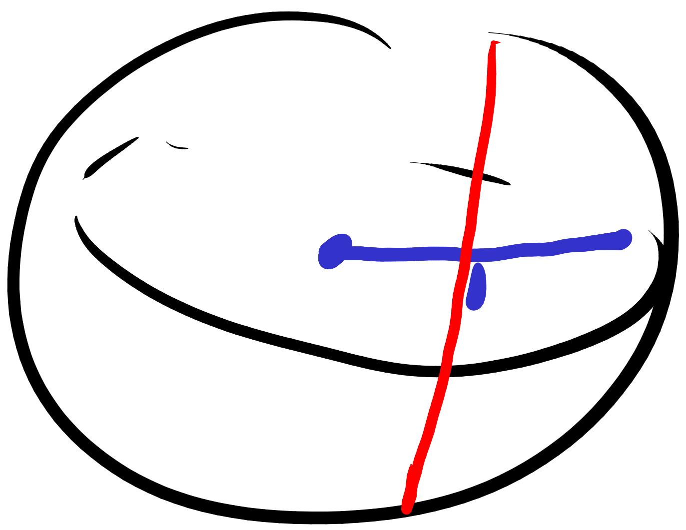
$$\int_0^4 \int_0^h r^3 h \, \frac{1}{2} \sin^2\theta \Big|_0^{2\pi} dr\,dh = \int_0^4 \int_0^h 0 \, dr\,dh = 0.$$

Q Volume of the unit sphere

$$\int_{-1}^{1}\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}}\int_{-\sqrt{1-y^2-z^2}}^{\sqrt{1-y^2-z^2}} 1 \, dx\,dy\,dz$$



$$\int_{0}^{2\pi}\int_{0}^{1}\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} 1\cdot r \, dh\,dr\,d\theta$$

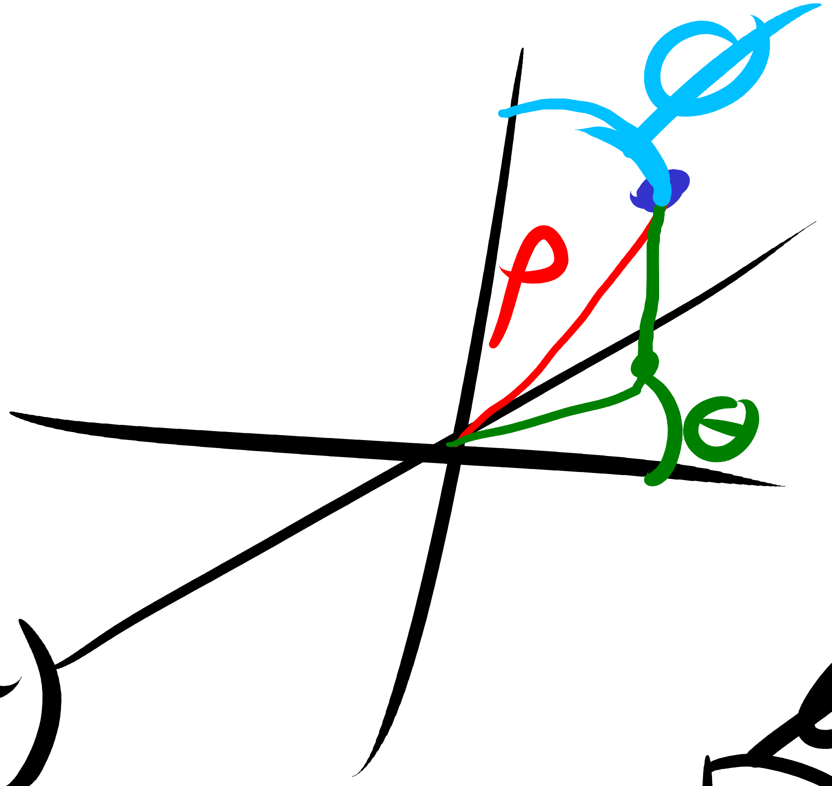


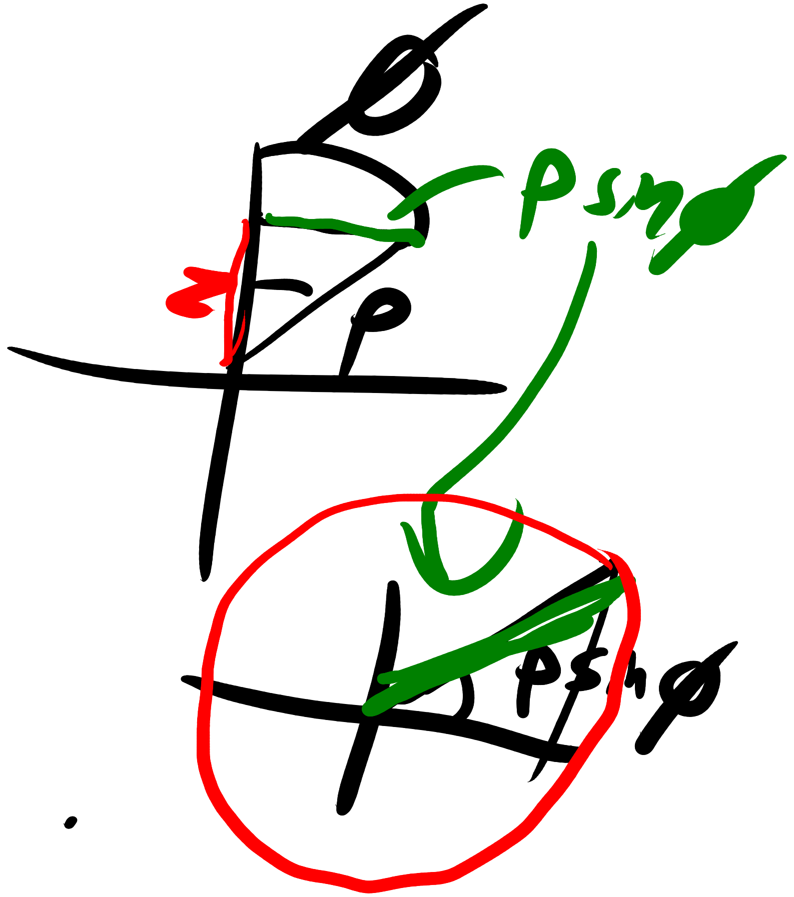
Spherical Coordinates

ρ = distance to origin ≥ 0

θ = angle in xy plane $\in [0, 2\pi)$

ϕ = angle from xy plane $\in [0, \pi]$





$$\rho^2 = x^2 + y^2 + z^2$$

$$z = \rho \cos\phi$$

$$x = \rho \sin\phi \cos\theta$$

$$y = \rho \sin\phi \sin\theta.$$

Integral formula

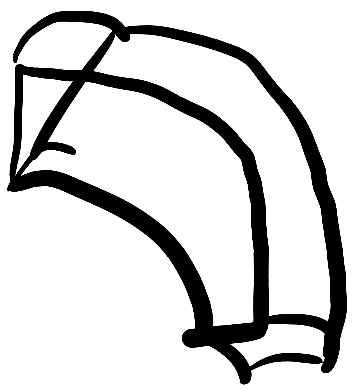
$d\rho \, d\theta \, d\phi$

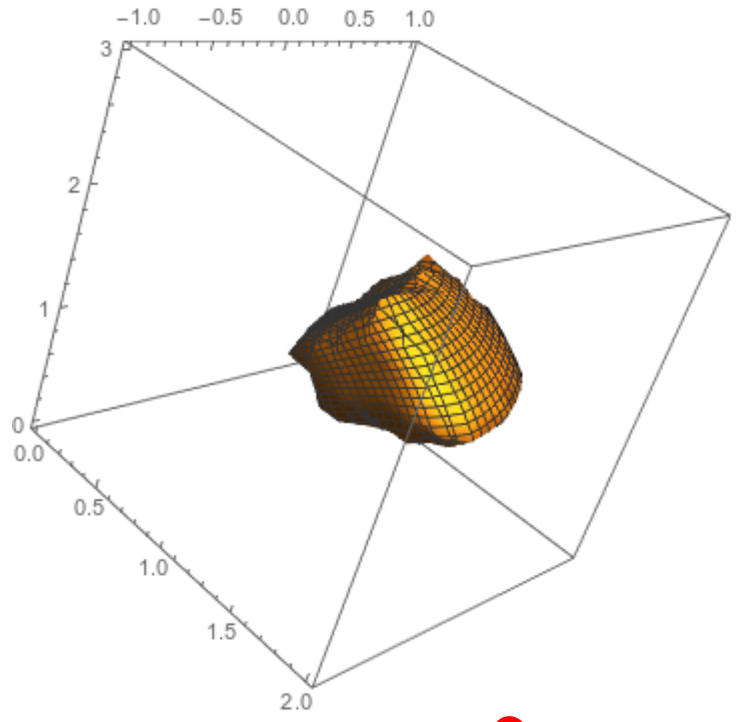
thickness $d\rho$

'height' $\rho \, d\phi$

'width' $\rho \sin\phi \, d\theta$

$\iiint \quad \rho^2 \sin\phi \, d\phi \, d\theta \, d\rho$





Volume of unit sphere?

$$\int_0^1 \int_0^{2\pi} \int_0^{\pi} 1 \cdot \rho^2 \sin\phi \, d\phi \, d\theta \, d\rho$$

$$= \int_0^1 \int_0^{2\pi} \rho^2 \left(-\cos\phi\right)\Big|_0^{\pi} d\theta \, d\rho = \int_0^1 \int_0^{2\pi} 2\rho^2 \, d\theta \, d\rho$$

$$= \int_0^1 4\pi\rho^2 \, d\rho = 4\pi \frac{\rho^3}{3}\Big|_0^1 = \frac{4}{3}\pi$$

mass of upper hemisphere radius 3, density $\rho\cos^2\theta$

$$\int_0^3 \int_0^{2\pi} \int_0^{\pi/2} \rho\cos^2\theta \, \rho^2 \sin\phi \, d\phi \, d\theta \, d\rho$$

$$= \frac{81\pi}{4}.$$

§6 Parametrization and vector fields.

$f: \mathbb{R} \to \mathbb{R}$ scalar fn

$f: \mathbb{R}^n \to \mathbb{R}$ multivariable fn

$f: \mathbb{R} \to \mathbb{R}^n$ Parametrized curve

$f: \mathbb{R}^n \to \mathbb{R}^n$ Vector field