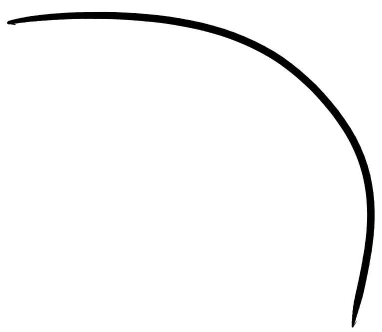
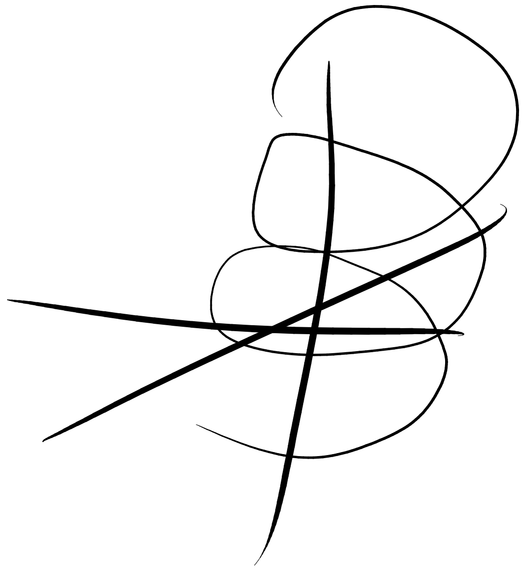
§6.1 Curves and Motion

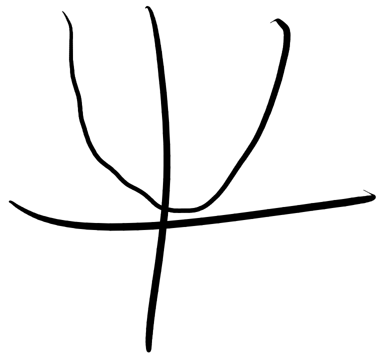




Dfn: $\vec{r} : \mathbb{R} \to \mathbb{R}^3$
is a parametrization
of a curve

Components
$$\vec{r}(t) = (x(t), y(t), z(t))$$

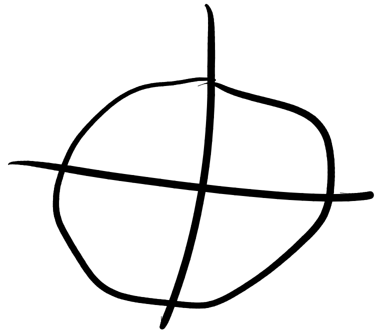
$y = x^2$



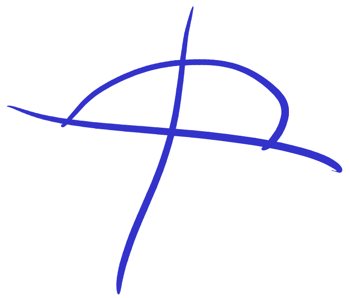
$\vec{r}(t) = (t, t^2)$

$\vec{G}(t) = (t, f(t))$

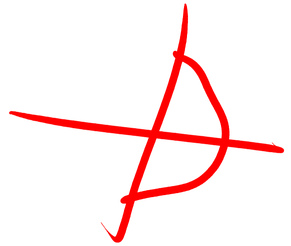
circle of radius 1



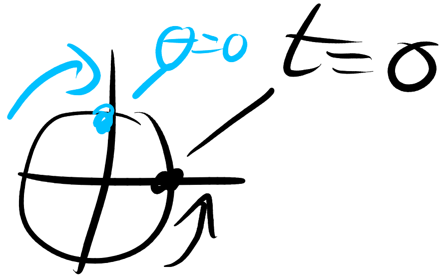
$\vec{r}(t) = (t, \sqrt{1-t^2})$



$\vec{r}(t) = (\sqrt{1-t^2}, t)$



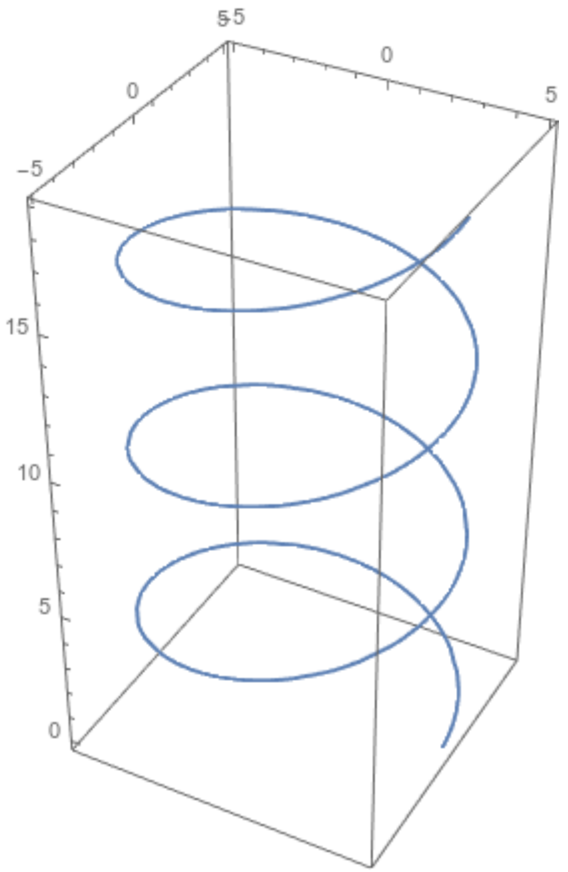
$\vec{r}(t) = (\cos t, \sin t)$



$r(\theta) = (\sin t, \cos t)$

$\vec{r}(t) = (5\cos t, 5\sin t, t)$

Spiral/helix



parametrize line through $(1,3,5)$
direction $2\vec{i} - \vec{j} + 3\vec{k}$

$$\vec{r}(t) = (1,3,5) + t\,(2,-1,3)$$

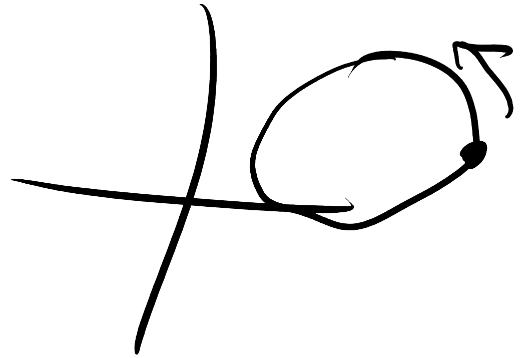
General formula is

$$\vec{r}(t) = \vec{r_0} + t\,\vec{v}$$

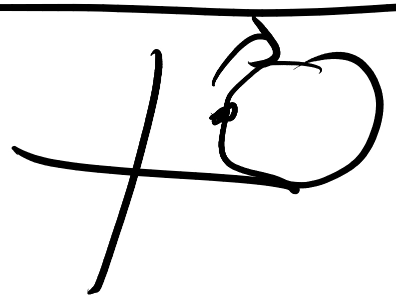
$$y = b + x m$$

Circle, radius 2, centered at (3, 2)

$$\vec{r}_0(t) = (2\cos t, 2\sin t) + (3, 2)$$

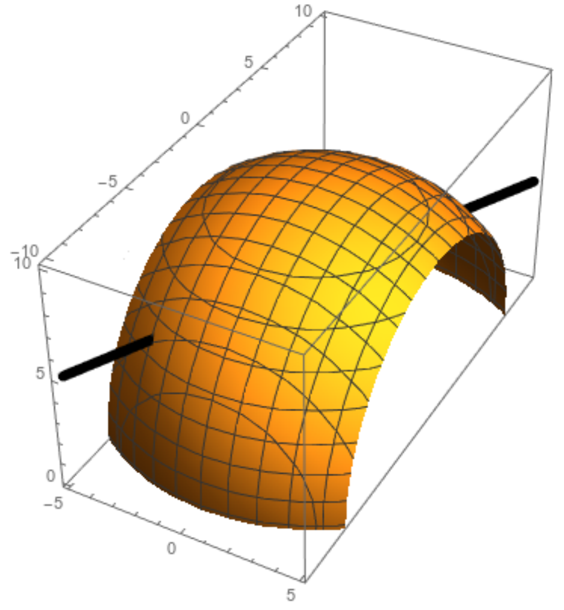


$$= (3 + 2\cos t, 2 + 2\sin t)$$



$$\vec{r}(t) = \left(2\sin 2t - \frac{\pi}{2}, 2\cos 2t - \frac{\pi}{2}\right) + (3, 2)$$

Where does curve intersect a surface?



$\vec{r}(t) = (t, 2t, t+3) = (0,0,3) + t(1,2,1)$

Sphere radius 9 centered at O

$$x^2+y^2+z^2=81$$

$$t^2+(2t)^2+(t+3)^2=81$$

$$6t^2+6t=72$$

$$(t+4)(t-3)=0$$

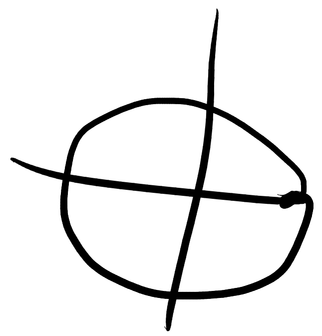
line hits sphere at time $t=-4$, $t=3$

$\vec{r}(-4) = (-4,-8,-1)$

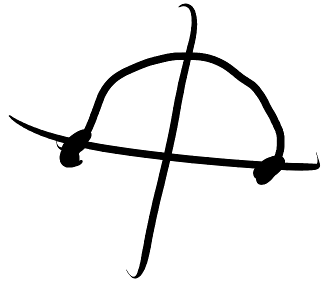
$\vec{r}(3) = (3,6,6)$

Parametrize a semicircle?

$\vec{r}(t) = (\cos t, \sin t)$ for $t \in [0, 2\pi]$

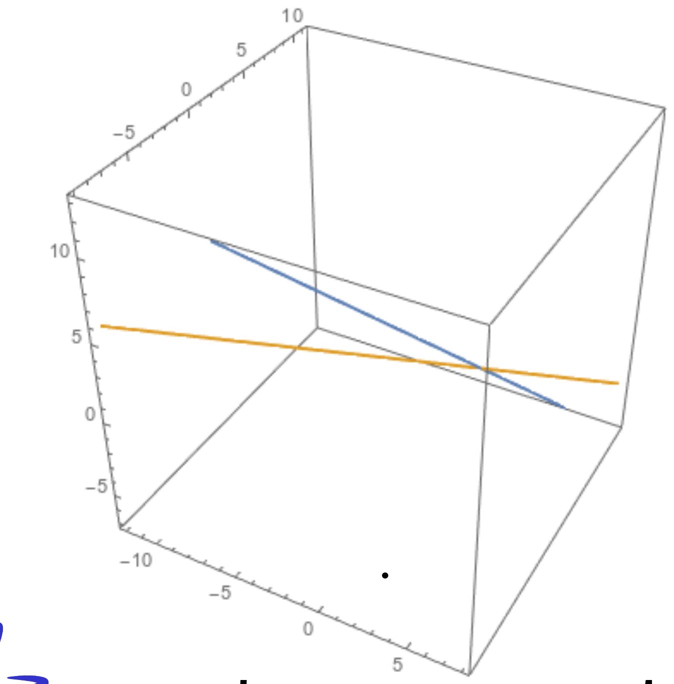


$\vec{s}(t) = (\cos t, \sin t)$ for $t \in [0, \pi]$



$\vec{r}_1 = (t, 1+2t, 3-2t)$

$\vec{r}_2 = (-2-2t, 1-2t, 1+t)$



$t = -2-2t \qquad t = -2/3$

$1+2t = 1-2t \qquad t = 0$

$3-2t = 1+t \qquad t = 2/3$

$t_1 = -2-2t_2 \qquad t_1 = 2t_1 - 2$

$1+2t_1 = 1-2t_2 \qquad t_2 = -t_1$

$3-2t_1 = 1+t_2 \qquad 3-2t_1 = 1-t_1$

$\Rightarrow t_1 = 2, \; t_2 = -2$

paths intersect $(2, 5, -1)$

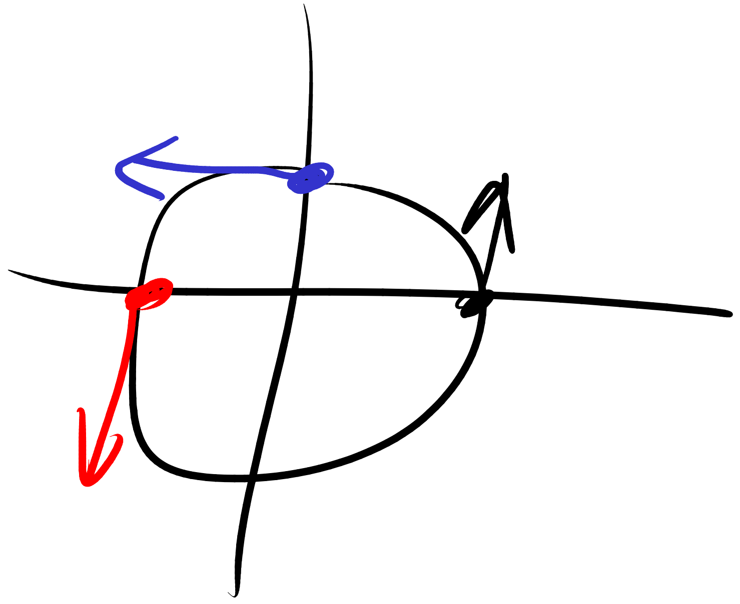
Dfn: The velocity of an object path $\vec{r}(t)$ is

$$\vec{v}(t) = \vec{r}\,'(t) = \frac{d\vec{r}}{dt} = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}$$

Prop: $r'(t) = (x'(t), y'(t), z'(t))$

Circle $\vec{r}(t) = (\cos(t), \sin(t))$

$\vec{r}'(t) = (-\sin(t), \cos(t))$



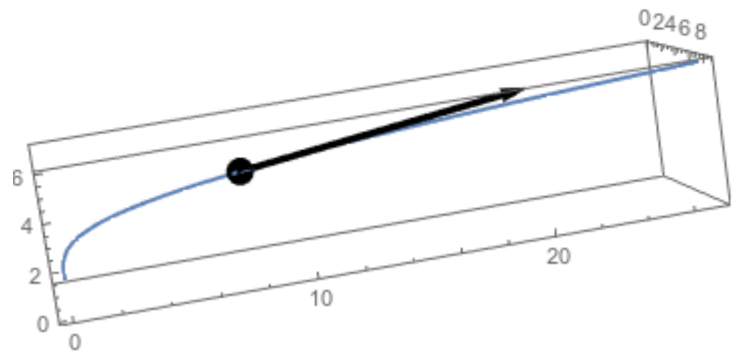
-1 $\quad$ 0

0 $\quad$ -1

$\vec{r}(t) = (t^2, t^3, 2t)$

$\vec{r}\,'(t) = (2t, 3t^2, 2)$

T line @ $t = 2$



$\vec{r}\,'(2) = (4, 12, 2)$

$\vec{r}(2) + t\,\vec{r}\,'(2) = (4, 8, 4) + t(4, 12, 2)$

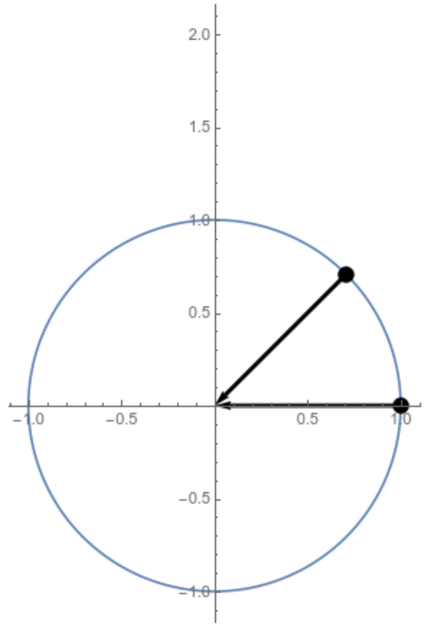
Dfn: Acceleration

$$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t) = \lim_{h \to 0} \frac{r'(t+h) - r'(t)}{h}$$

$$\vec{a}(t) = (x''(t), y''(t), z''(t))$$

$$\vec{r}(t) = (\cos(t), \sin(t))$$

$$\vec{r}\,''(t) = (-\cos(t), -\sin(t))$$



§6.2 Surfaces

Dfn: a parametrization of a surface is a fn

$\bar{r}: \mathbb{R}^2 \to \mathbb{R}^n$

$$\bar{r}(s,t) = \big(x(s,t),\, y(s,t),\, z(s,t)\big)$$

graph of fn $z = f(x,y)$ parametrized by

$$\bar{r}(s,t) = (s,\, t,\, f(s,t))$$