§6.2 Surfaces

$$\vec{r}: \mathbb{R}^2 \to \mathbb{R}^n$$

$$\vec{r}(s,t) = (x(s,t), y(s,t), z(s,t))$$

Curve

$$\vec{r}: \mathbb{R} \to \mathbb{R}^n$$

$$\vec{r}(t) = (x(t), y(t), z(t))$$

Ex: sphere of radius 5

$$\vec{r}(\theta, \phi) = (5\sin\phi\cos\theta,\ 5\sin\phi\sin\theta,\ 5\cos\phi)$$

centered at $(3,2,4)$

ellipsoid

$$\vec{r}_2(\theta, \phi) = (3+5\sin\phi\cos\theta,\ 2+\overset{10}{\cancel{5}}\sin\phi\sin\theta,\ 4+5\cos\phi)$$

cylinder, radius 1, centered at 0, opens in z-axis

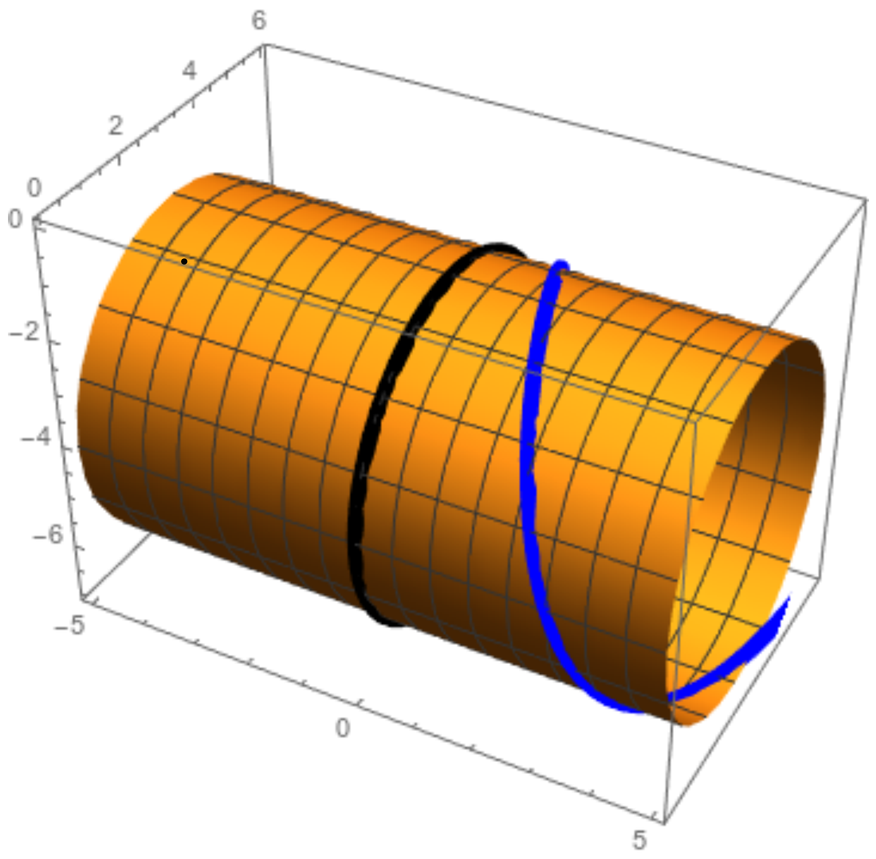
$$\vec{r}(\theta, h) = (\cos\theta, \sin\theta, h)$$

cylinder radius 3, centered at line $y=3$, $z=-4$ opening in x-axis

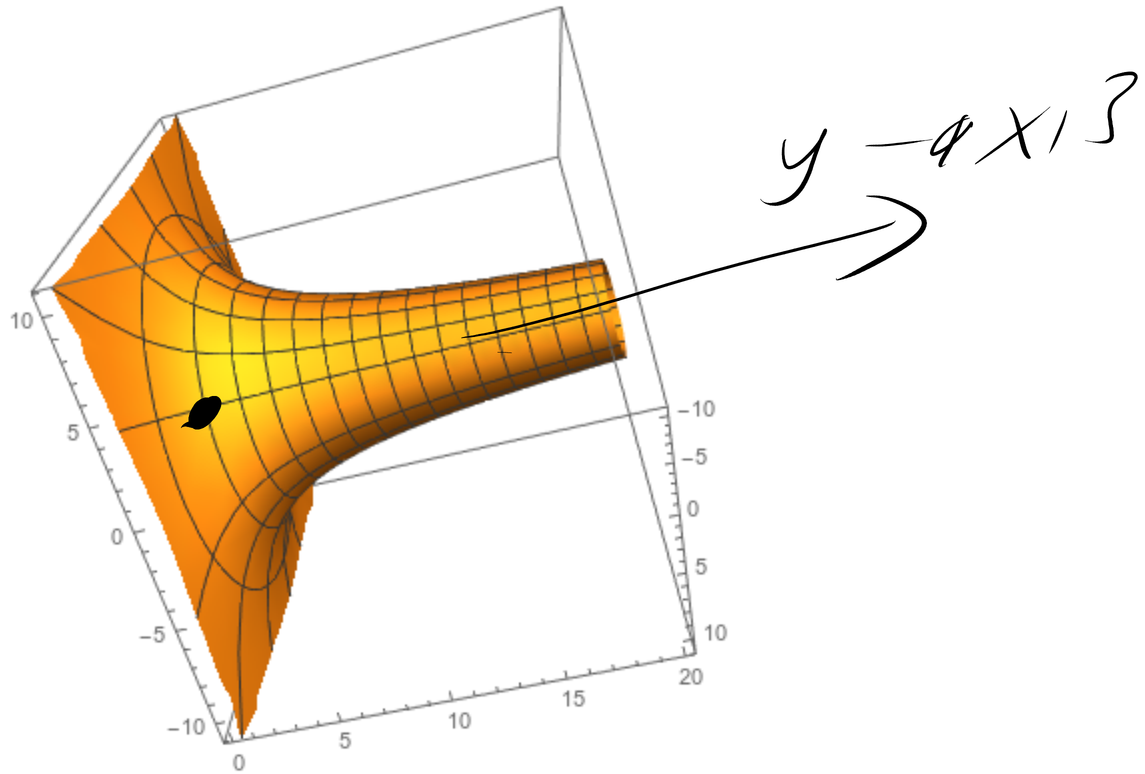
$$\vec{r}(\theta, s) = (s, 3\cos\theta + 3, 3\sin\theta - 4)$$

$$\vec{r}(\theta, s) = (s-500, 3\sin\theta + 3, 3\cos\theta - 4)$$

$$\vec{r}(\theta, s) = (s+\theta, 3\sin\theta + 3, 3\cos\theta - 4)$$



Trumpetshape
bell @ origin
narrowing along y-axis
radius is $f(y) = \frac{10}{\sqrt{y}}$



$\vec{r}(s,t) = \left(\frac{10}{\sqrt{s}}\cos t,\ s,\ \frac{10}{\sqrt{s}}\sin t\right)$ for $s \in (0, \infty)$

$\vec{r}(4,t) = (5\cos t, 4, 5\sin t)$ circle in xz plane $y = 4$

§6.3 Change of coordinates

$\vec{r}(s,t) = (s,t)$

$\vec{r}(s,t) = (t,s)$

$\vec{r}(s,t) = (3s, s-t)$

$\vec{r}(s,t) = (s\cos t, s\sin t)$

polar coordinates

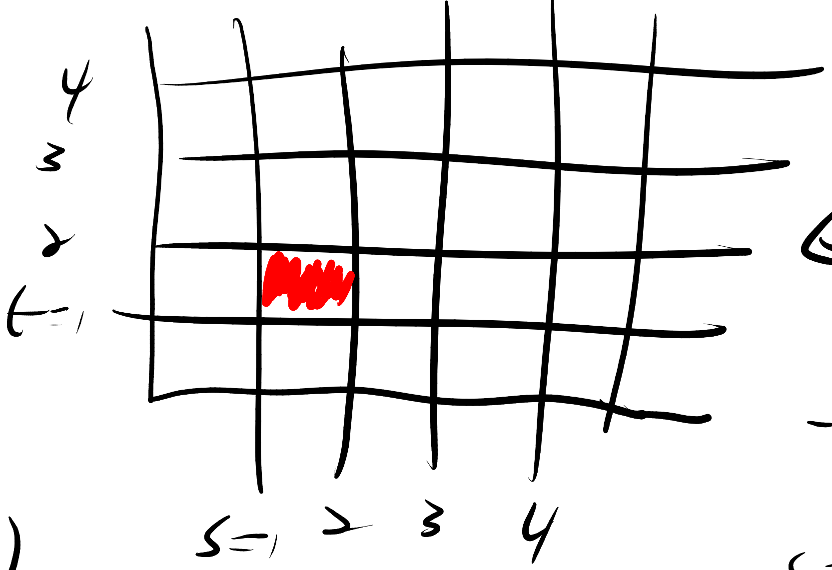
Integrals!

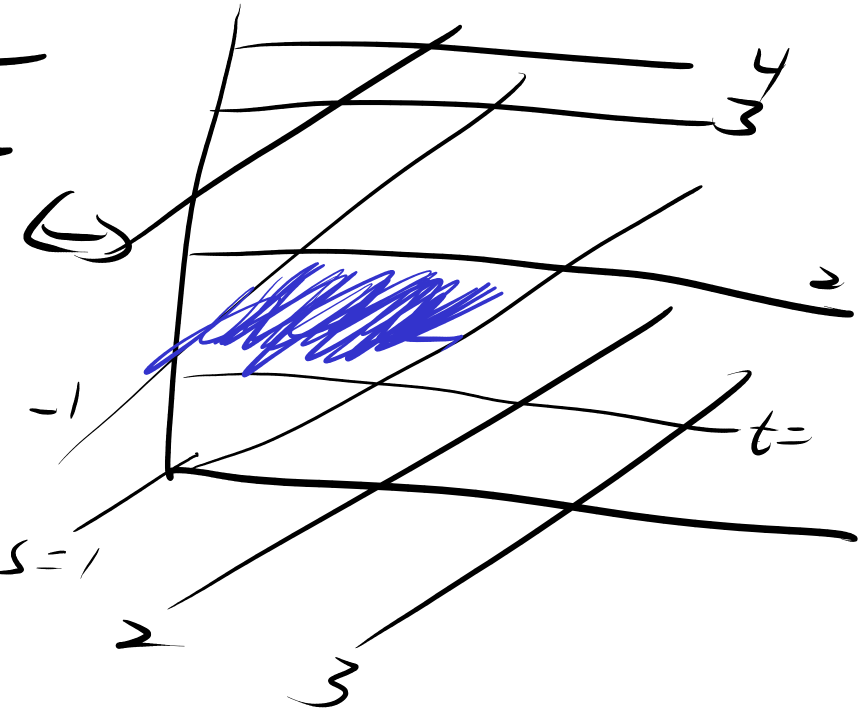
Integrals!

$\vec{r}(s,t) = (x(s,t), y(s,t))$

rect in s-t coords

$(s,t), (s+\Delta s, t), (s, t+\Delta t), (s+\Delta s, t+\Delta t)$



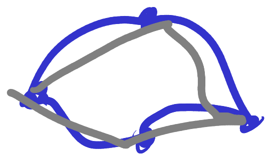


$\downarrow$

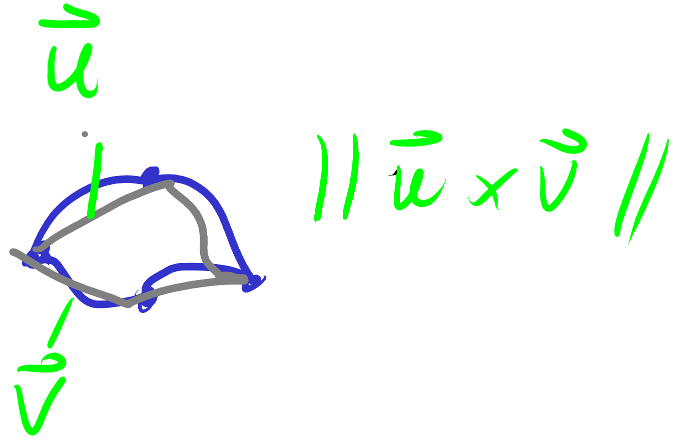
$(x(s,t), y(s,t))$

$\downarrow$

$(x(s+\Delta s, t+\Delta t), y(s+\Delta s, t+\Delta t))$



Approx w/ parallelogram.



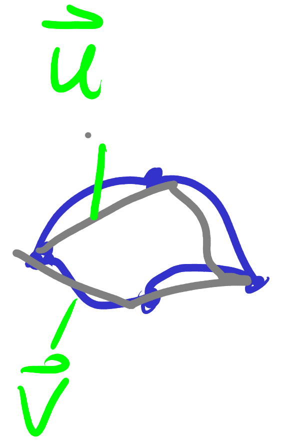
Area of ▱ w/ sides given by $\vec{u}$, $\vec{v}$ is $\|\vec{u}\times\vec{v}\|$

$\vec{u} = (x(s+\Delta s, t), y(s+\Delta s, t)) - (x(s,t), y(s,t))$

$$\vec{u} = (x(s+\Delta s, t), y(s+\Delta s, t)) - (x(s,t), y(s,t))$$

$$= \frac{\vec{r}(s+\Delta s, t) - \vec{r}(s,t)}{\Delta s} \Delta s = \frac{\partial \vec{r}}{\partial s} \Delta s$$

$$= \frac{\partial x}{\partial s} \Delta s \, \vec{i} + \frac{\partial y}{\partial s} \Delta s \, \vec{j}$$



$$\vec{v} = \frac{\partial x}{\partial t} \Delta t \, \vec{i} + \frac{\partial y}{\partial t} \Delta t \, \vec{j}$$

$$\|\vec{u} \times \vec{v}\| = \left| \frac{\partial x}{\partial s} \frac{\partial y}{\partial t} - \frac{\partial x}{\partial t} \frac{\partial y}{\partial s} \right| \Delta s \Delta t$$

$\approx$ area of my s, t "rectangle"

Dfn: the <u>Jacobian</u> of a fn

its the deteminant of thematrix of partials

$$\frac{\partial(x,y)}{\partial(s,t)} = \frac{\partial x}{\partial s}\frac{\partial y}{\partial t} - \frac{\partial x}{\partial t}\frac{\partial y}{\partial s} = \begin{vmatrix} \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \end{vmatrix}$$

area of ▱ $\simeq \left|\frac{\partial(x,y)}{\partial(s,t)}\right| \Delta s \Delta t$

$$\int_R f(x,y)\,dA = \lim \sum f(x^*,y^*)\left|\frac{\partial(x,y)}{\partial(s,t)}\right| \Delta s\,\Delta t$$

$$= \lim \sum \underbrace{\overbrace{f(x(s^*,t^*),y(s^*,t^*))}^{h} \overbrace{\left|\frac{\partial}{\partial}\right|}^{\text{Area}}}_{h} \underline{\Delta s\,\Delta t}$$

$$= \int \overline{f(x(s,t),y(s,t))} \left|\frac{\partial(x,y)}{\partial(s,t)}\right| \overbrace{ds\,dt}^{\text{Area}}$$

Polar

$x = r\cos\theta$

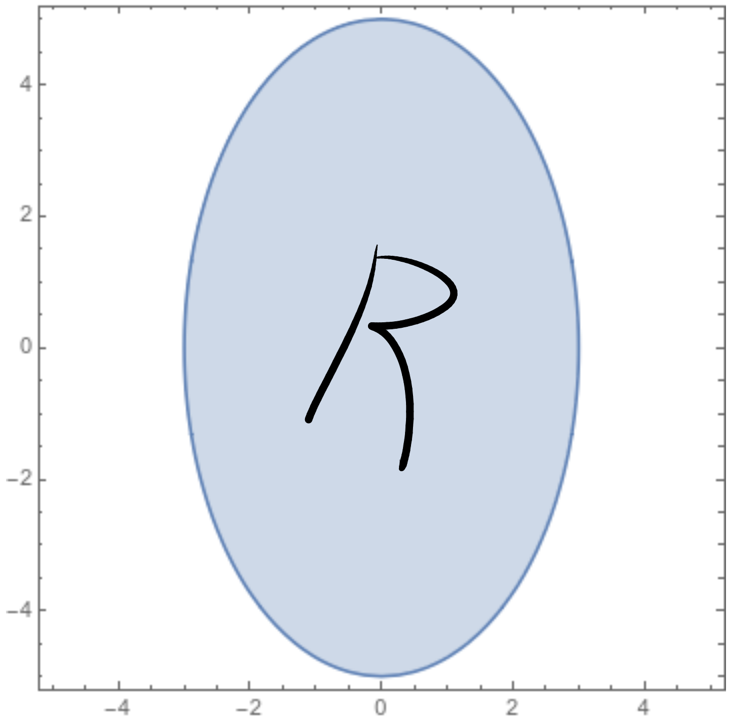
$y = r\sin\theta$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$$= r\cos^2\theta + r\sin^2\theta = r(\cos^2\theta + \sin^2\theta) = 1.$$

$$= \int\int f(r\cos^2\theta + r\sin^2\theta)\, dr\, d\theta$$

Find area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



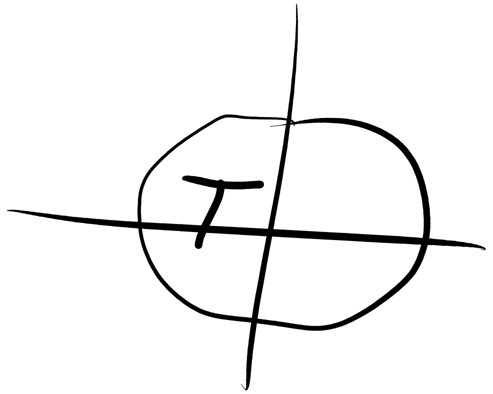
$x=as, \quad y=bt$

$r(s,t)=(as, bt)$

$\frac{(as)^2}{a^2}+\frac{(bt)^2}{b^2}=1$

$s^2+t^2=1$

$$\frac{\partial(x,y)}{\partial(s,t)} = \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} = ab$$



$$\int_R 1\,dx\,dy = \int_T 1\cdot ab\,ds\,dt = ab\int_T 1\,ds\,dt$$

$$= \pi ab.$$

$$\int_R x+y \; dA$$