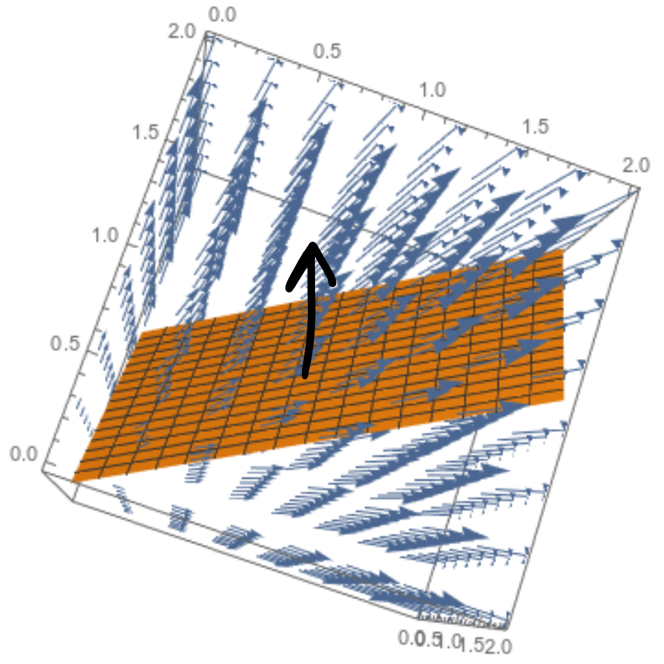
$$\iint_S \vec{F}\cdot d\vec{A} = \int_a^b \int_c^d \vec{F}(\vec{r}(s,t))\cdot(\vec{r}_s \times \vec{r}_t)\, ds\, dt$$

1) S graph of a fn
2) S cylinder
3) S sphere

$$\vec F(x,y,z) = x\vec i + y\vec j$$

$$\vec r(s,t) = (2s,\ s+t,\ 1+s-t) \qquad 0\le s\le 1,\ 0\le t\le 1$$

oriented down



$$\vec r_s(s,t) = (2,1,1) \qquad \vec r_t(0,1,-1)$$

$$\vec r_s\times\vec r_t = \begin{vmatrix} \vec i & \vec j & \vec k \\ 2 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -2\vec i + 2\vec j + 2\vec k$$

$$\int_0^1\int_0^1 (2s,\ s+t,\ 0)\cdot(2,-2,-2)\,ds\,dt = \int_0^1\int_0^1 2s-2t\,ds\,dt$$

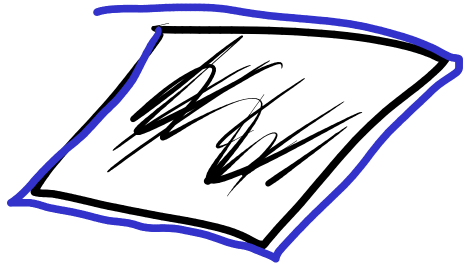
$$= \int_0^1 s^2-2st\Big|_0^1 dt = \int_0^1 1-2t\,dt = t-t^2\Big|_0^1 = 0.$$

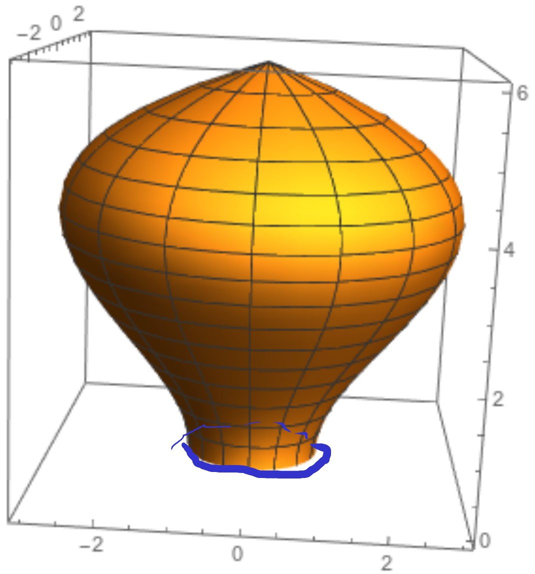
§8.3 Stokes's Theorem

Stokes'

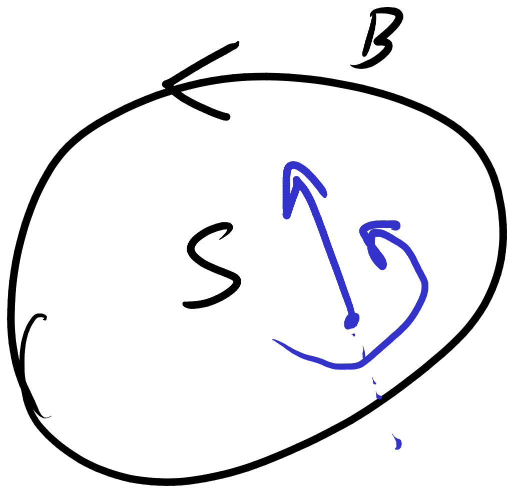
~~Stoke's~~

Boundary of a surface



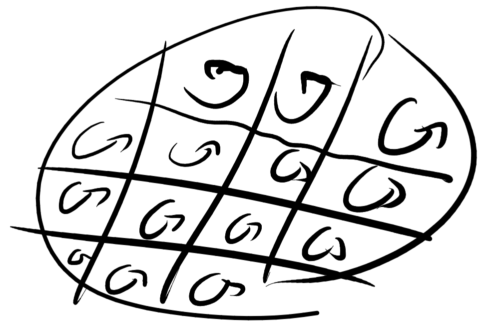


Want to match orientation of surface w/ orientation of bdry.



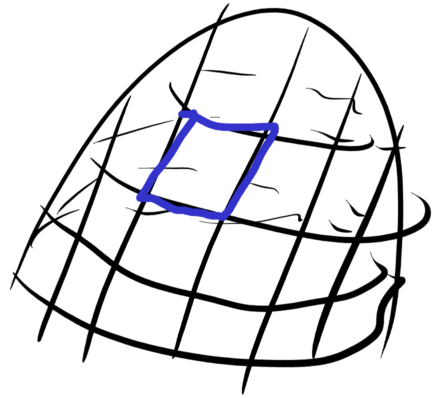
Thm: S smooth oriented surface, bdry C matching orientation

$\vec{F}$ a smooth VF on region containing S and C. Then



Greens: $\int_C \vec{F}\cdot d\vec{r} = \int_R (\nabla\times\vec{F})\cdot \underbrace{\boxed{\vec{k}\, dA}}_{d\vec{A}}$

Stokes: $\int_C \vec{F}\cdot d\vec{r} = \int_S \nabla\times\vec{F}\cdot d\vec{A}$



"Proof"/ chop surface into little pieces.

circulation around piece is $\int_{\Delta C} \vec{F}\cdot d\vec{r}$

but it's also $\approx \nabla\times\vec{F}\cdot \Delta\vec{S}$

$\vec{F}(x,y,z) = -2y\,\vec{i} + 2x\,\vec{j}$

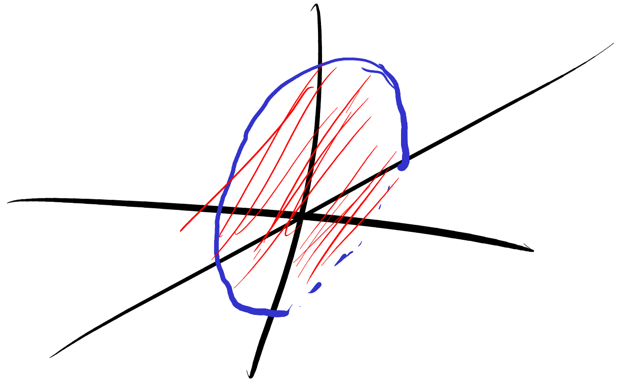
C: circle parallel to yz plane centered at x axis

$\int_C \vec{F}\cdot d\vec{r} = \int_S \nabla\times\vec{F}\cdot d\vec{A}$

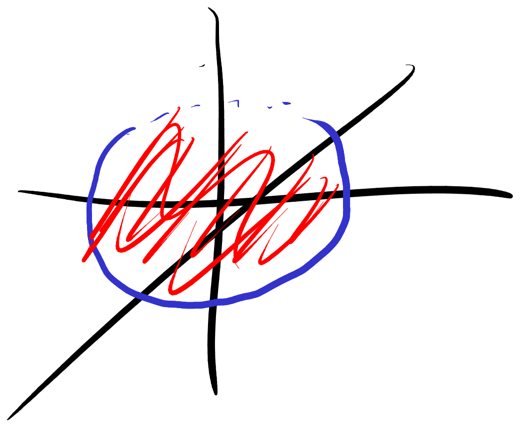
$\nabla\times\vec{F} = 4\vec{k}$

$d\vec{A} = \_\_\,\vec{i}$

so $\nabla\times\vec{F}\cdot d\vec{A}$



What if circle parallel to xy plane instead?



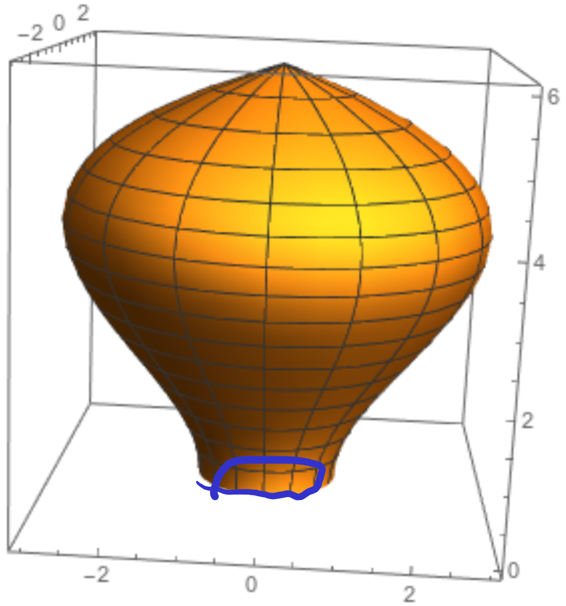
$\nabla\times\vec{F} = 4\vec{k}$

$\vec{A} = (\text{area of circle})\,\vec{k}$

$d\vec{A} = (\text{area of small piece of circle})\,\vec{k}$

$\int_C \vec{F}\cdot d\vec{r} = \int 4\vec{k}\cdot\vec{k}\,dA = 4\pi r^2$

Surface of a lightbulb
w/ base $x^2+y^2=1$



$$\vec{F}(x,y,z) = e^{z^2-2z}x\,\vec{i} + (\sin(xyz)+y+1)\,\vec{j} + e^{z^2}\sin(z^2)\,\vec{k}$$

$$r(\theta) = (\cos\theta, \sin\theta, 0)$$

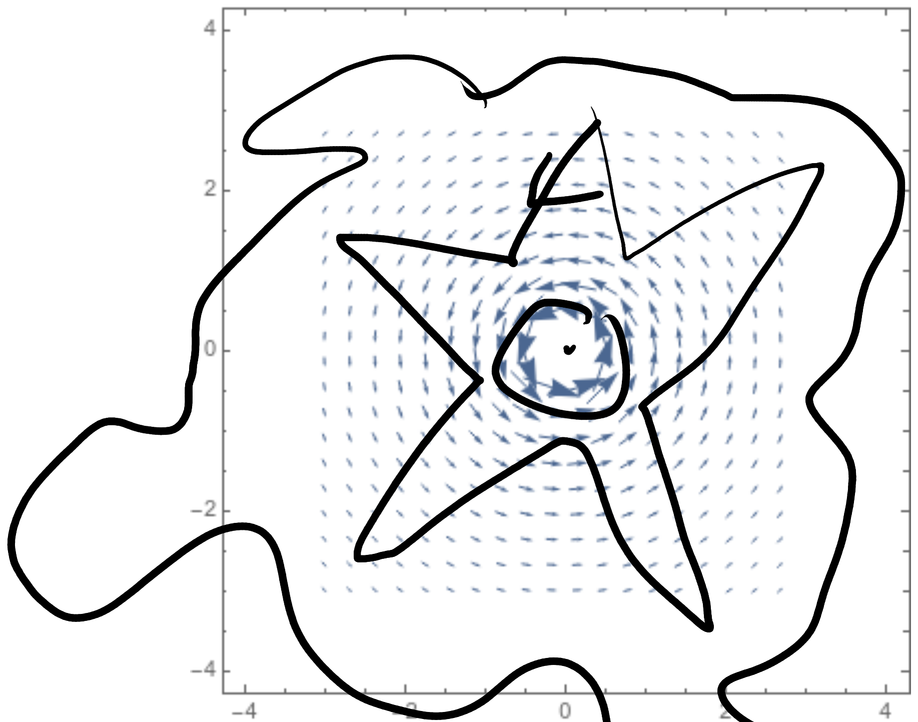
$$\int_S \nabla\times\vec{F}\cdot d\vec{A} = \int_C \vec{F}\cdot d\vec{r}$$

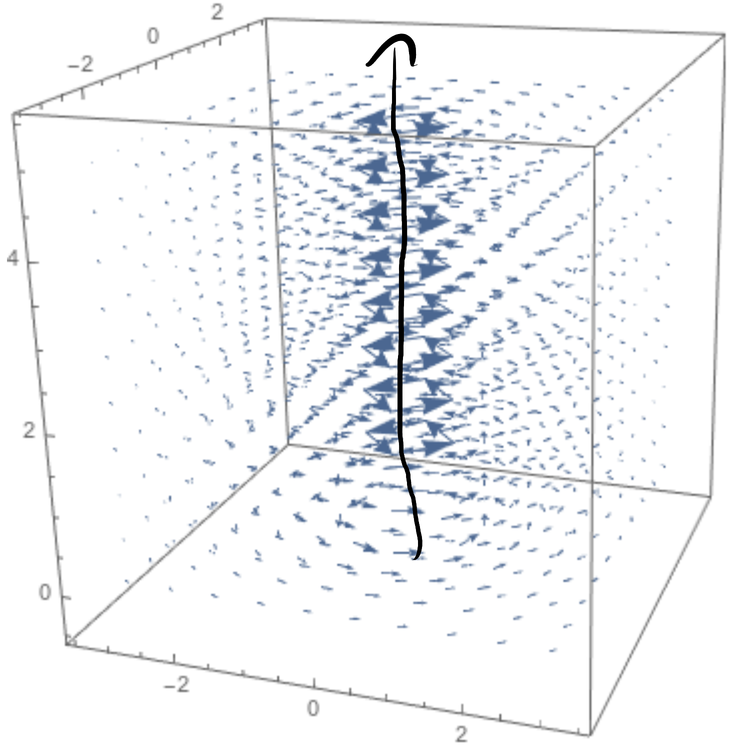
$$= \int_0^{2\pi} (\cos\theta, \sin\theta+1, 0)\cdot(-\sin\theta, \cos\theta, 0)\, d\theta$$

$$= \int_0^{2\pi} \cancel{-\sin\theta\cos\theta + \sin\theta\cos\theta} + \cos\theta\, d\theta$$

$$= \sin\theta\,\Big|_0^{2\pi} = 0.$$

$$\vec{B}(x,y,z) = \frac{-y\vec{i} + x\vec{j}}{x^2+y^2}$$



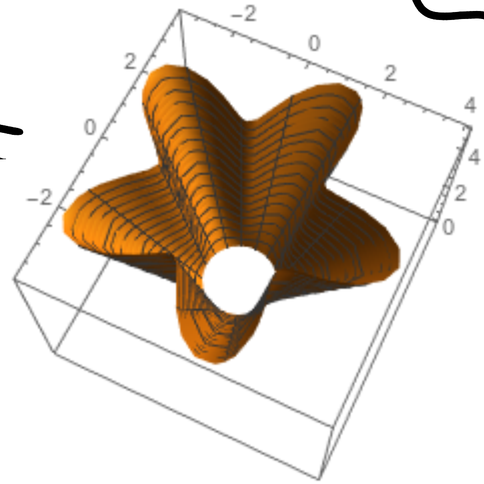


$$\nabla \times \vec{B} = \vec{0}$$

compute circulation around $\star$

$$\int_C \vec{B}\cdot d\vec{r}$$

$S =$



Boundary of S is $\overset{\star}{C} - \overset{\text{circle}}{C_2}$

$$0 = \int_S \nabla \times \vec{B}\cdot d\vec{A} = \int_C \vec{B}\cdot d\vec{r} - \int_{C_2} \vec{B}\cdot d\vec{r}$$

So $\int_C \vec{B}\cdot d\vec{r} = \int_{C_2} \vec{B}\cdot d\vec{r} = \int_0^{2\pi} (-\sin\theta, \cos\theta)\cdot(-\sin\theta, \cos\theta)\, d\theta = 2\pi.$