Math 114 Fall 2017
Calculus I HW 1 Solutions
Updated Sept 2
Due Wednesday, September 6

1. (a) Find two real numbers that solve $x^2 + 7x + 5 = 0$. **Solution:** By the quadratic formula, we have
$$x = \frac{-7 \pm \sqrt{49 - 20}}{2} = \frac{-7 \pm \sqrt{29}}{2}.$$
   (b) Factor $x^3 - 27$.
   Solution: By the difference of cubes formula, we have $x^3 - 27 = (x^2 + 3x + 9)(x - 3)$. We can check our work by multiplying this out:
$$(x^2 + 3x + 9)(x - 3) = x^3 + 3x^3 + 9x - 3x^3 - 9x - 27 = x^3 - 27.$$

2. Based on the graphs below, estimate the following limits:
   (a) $\lim_{x\to 1} f(x)$
   (b) $\lim_{x\to -2} g(x)$
   (c) $\lim_{x\to 1} h(x)$
   (d) $\lim_{x\to 1} j(x)$

   Solution:
   (a) 0
   (b) -6
   (c) 2
   (d) 3

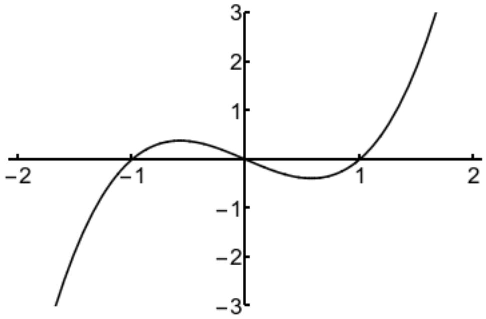

Figure 1: $f(x)$

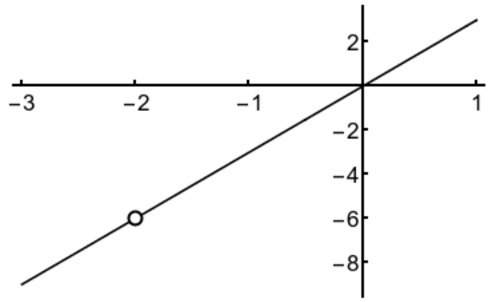

Figure 2: $g(x)$

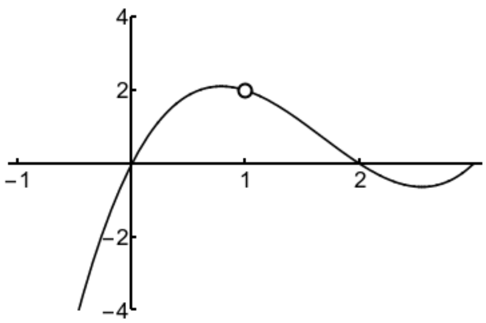

Figure 3: $h(x)$

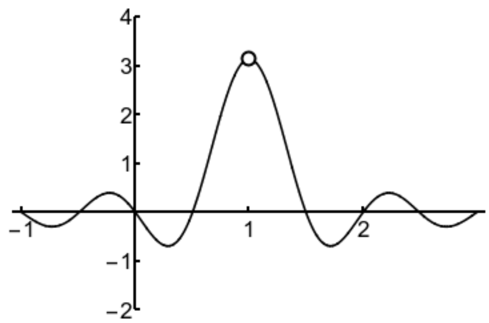

Figure 4: $j(x)$

3. If $|f(x)| \leq |x|$ and $|g(x)| \leq 7 + x^2$, what can we say about $|f(x) + g(x)|$?

   Solution: By the triangle inequality we know that

$$|f(x) + g(x)| \leq |f(x)| + |g(x)| \leq |x| + 7 + x^2.$$

   By the reverse triangle inequality, we know that

$$|f(x) + g(x)| \geq |f(x)| - |g(x)| \geq |f(x)| - 7 - x^2$$
$$|g(x) + f(x)| \geq |g(x)| - |f(x)| \geq |g(x)| - |x|.$$

   (We *cannot* replace the $|f(x)|$ with $|x|$ in the first inequality, because knowing that $|f(x)| \leq |x|$ cannot tell us anything about what $|f(x)|$ is *bigger* than. Similarly with $|g(x)|$ in the second inequality).

4. If $|f(x)| \geq 7$ and $|g(x)| \leq 3$, what can we say about $|f(x) + g(x)|$?

   Solution: By the triangle inequality we know that

$$|f(x) + g(x)| \leq |f(x)| + |g(x)| \leq |f(x)| + 3.$$

   Here we cannot do anything with the $|f(x)|$ term because we don't have an upper bound on it.

   By the reverse triangle inequality, we know that

$$|f(x) + g(x)| \geq |f(x)| - |g(x)| \geq 7 - 3 = 4|g(x) + f(x)| \qquad \geq |g(x)| - |f(x)|.$$

This time we can replace $|f(x)|$ in the first inequality, since we know how *small* $f(x)$ is, not how big it is. We can't replace either $|g(x)|$ or $|f(x)|$ in the second inequality, so it's not really useful at all.

5. $\star$

   (a) Find a pair of real numbers x and y such that $|x+y| < |x| + |y|$.

   Solution: There are many correct solutions to all of these problems, but one example is $|1 + (-1)| < |1| + |-1|$.

   (b) Find a pair of real numbers x and y such that $|x+y| = |x| + |y|$.

   Solution: $|1 + 1| = |1| + |1|$.

   (c) Find a pair of real numbers x and y such that $|x+y| > x + y$.

   Solution: $|(-1) + (-1)| > (-1) + (-1)$.

6. $\star$

   (a) Find a pair of real numbers x and y such that $|x+y| > |x| - |y|$.

   Solution: $|1 + 1| > |1| - |1|$.

   (b) Find a pair of real numbers x and y such that $|x+y| = |x| - |y|$.

   Solution: $|1 + (-1)| = 1 - |-1|$.

   (c) Find a pair of real numbers x and y such that $|x+y| < x - y$.

   Solution: $|1 + (-1)| < 1 - (-1)$.

7. Let $f(x) = 2x + 1$, and let $L = 3$.

   (a) Suppose we have an error margin of $\epsilon = 1/10$, that is, we would like the distance between $f(x)$ and L to be less than $1/10$. What open interval does x need to be in to make this happen?

   (b) Now suppose our error margin is $\epsilon = 1/50$. Give an open interval for x so that the distance between $f(x)$ and L is less than $1/50$ for every x in the interval.

   Solution:

   (a) We have that $|f(x) - L| < \epsilon$, and thus $|2x - 2| < 1/10$, and thus $|x - 1| < 1/20$. This tells us that the distance from x to 1 is less than $1/20$, so x must be in the interval $(1 - 1/20, 1 + 1/20) = (.95, 1.05)$.

   Alternatively, we can write that $3 - \epsilon < f(x) < 3 + \epsilon$, and thus $2.9 < 2x + 1 < 3.1$. Solving this gives $1.9 < 2x < 2.1$ and thus $.95 < x < 1.05$.

   (b) We have that $|f(x) - L| < \epsilon$ and thus $|2x - 2| < 1/50$, and thus $|x - 1| < 1/100$. Thus x is in the interval $(1 - 1/100, 1 + 1/100) = (.99, 1.01)$.

   Alternatively, again, we can take $2.98 < 2x + 1 < 3.02$, so $1.98 < 2x < 2.02$, so $.99 < x < 1.01$.