Math 114 Fall 2017
Calculus I HW 4 Solutions
Due Wednesday, October 4

1. Let a and c be any constants. From the ϵ-δ definition, prove that $\lim_{x\to a} c = c$.

   Solution: Let $\epsilon > 0$ and let $\delta = 1$. Then if $0 < |x - a| < \delta$, we compute

   $$|f(x) - c| = |c - c| = 0 < \epsilon.$$

2. Explicitly naming the rule used in each step, calculate $\lim\limits_{x\to 0} x^2 - 3x + 5$

   Solution:

   $$\begin{aligned}
   \lim_{x\to 0} x^2 - 3x + 5 &= \lim_{x\to 0} x^2 - \lim_{x\to 0} 3x + \lim_{x\to 0} 5 && \text{additivity}\\
   &= \left(\lim_{x\to 0} x\right)^2 - 3\lim_{x\to 0} x + \lim_{x\to 0} 5 && \text{exponents, scalars}\\
   &= (0)^2 - 3\cdot 0 + 5 && \text{identity, constants}\\
   &= 0 - 0 + 5 = 5.
   \end{aligned}$$

3. Explicitly naming the rule used in each step, calculate $\lim\limits_{x\to 4} \sqrt{x} + \sqrt[3]{4+x}$

   Solution:

   $$\begin{aligned}
   \lim_{x\to 4} \sqrt{x} + \sqrt[3]{4+x} &= \lim_{x\to 4} \sqrt{x} + \lim_{x\to 4} \sqrt[3]{4+x} && \text{additivity}\\
   &= \sqrt{\lim_{x\to 4} x} + \sqrt[3]{\lim_{x\to 4} 4 + x} && \text{exponents}\\
   &= \sqrt{\lim_{x\to 4} x} + \sqrt[3]{\lim_{x\to 4} 4 + \lim_{x\to 4} x} && \text{additivity}\\
   &= \sqrt{4} + \sqrt[3]{4+4} && \text{identity, constants}\\
   &= 2 + 2 = 4.
   \end{aligned}$$

4. Explicitly naming the rule used in each step, calculate $\lim_{x\to 2} f(x)$ where

   $$f(x) = \begin{cases} x + 1 & x < 2 \\ x^2 - 1 & x > 2 \end{cases}$$

   Solution:

We need to calculate the two one-sided limits. We have

$$\begin{aligned}
\lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} x+1 && \text{Almost Identical Functions}\\
&= \lim_{x\to 2^-} x + \lim_{x\to 2^-} 1 && \text{Additivity}\\
&= \lim_{x\to 2^-} x + 1 && \text{Constants}\\
&= 2+1 = 3 && \text{Identity}\\
\lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} x^2 - 1 && \text{Almost Identical Functions}\\
&= \lim_{x\to 2^+} x^2 - \lim_{x\to 2^+} 1 && \text{Additivity}\\
&= \left(\lim_{x\to 2^+} x\right)^2 - \lim_{x\to 2^+} && \text{Exponents}\\
&= (2)^2 - \lim_{x\to 2^+} 1 && \text{Identity}\\
&= 4 - 1 = 3 && \text{Constants}
\end{aligned}$$

We see that $\lim_{x\to 2^+} f(x) = 3 = \lim_{x\to 2^-} f(x)$, so by agreement of the left and right limits, $\lim_{x\to 2} f(x) = 3$.

5. Stewart 1.6.20

6. Stewart 1.6.22

7. Stewart 1.6.24

8. By any means we have developed in class, compute $\lim_{x\to+\infty} x - \sqrt{x}$.

   Solution:

$$\begin{aligned}
\lim_{x\to+\infty} x - \sqrt{x} = \lim_{x\to+\infty} (x-\sqrt{x}) \cdot \frac{x+\sqrt{x}}{x+\sqrt{x}} &= \lim_{x\to+\infty} \frac{x^2 - x}{x+\sqrt{x}}\\
= \lim_{x\to+\infty} \frac{1-1/x}{1/x + 1/x^{3/2}}.
\end{aligned}$$

   The top goes to 1 and the bottom goes to 0, so this limit is some form of infinity. Since x is approaching $+\infty$, the top and bottom are both always positive, so the limit is $+\infty$.