Lab 5   Tuesday October 3

Trigonometry

1. Define a function `h[x_]:=Sin[1/x]`. What is `h[0]`? Plug it into Mathematica and see what answer you get.

2. Plug in about ten numbers getting close to 0. What happens?

3. Plot a graph of `h[x]` with domain $[-1, 1]$. What happens near zero? Try using smaller domains like $[-.1, .1]$ or $[-.01, .01]$.

4. Use the `Limit` command to compute the limit at 0. What do you think this means?

5. Define `j[x_]:=x * Sin[1/x]`. What do you think will happen near zero? Plug in some points near 0 to refine your guess.

6. Plot a graph of `j[x]` with domain $[-1, 1]$. What happens near zero? Plot more graphs with smaller domains.

7. Use the `Limit` command to compute the limit at 0.

8. Define `k[x_]:=Sin[x]/x`. What do you think will happen near zero? Plug in some points near 0 to refine your guess.

9. Plot a graph of `k[x]` with domain $[-\pi, \pi]$. What happens near zero? Plot more graphs with smaller domains. Also plot some larger domains. Do these pictures match your expectations?

10. Use the `Limit` command to compute the limit at 0.

The Squeeze Theorem

Soon we will study a principle called the "Squeeze Theorem" or the "Two Policemen Theorem", which allows us to compute the limit of a function we don't like by "trapping" or "squeezing" it between two functions which do. In the rest of this lab we'll visualize a few examples.

1. Earlier in this lab, we considered the function `x*Sin[1/x]` and its limit at zero. Now we want to generate better understanding of its behavior there. Plot `x * Sin[1/x]` on the same graph as `Abs[x]` and `-Abs[x]`. (Bonus: color-code the graph so `x*Sin[1/x]` is one color and the other two graphs are a different color, using the PlotStyle command).

2. Shrink the domain around the point $x = 0$ to see what happens.

3. What are the limits of $|x|$ and $-|x|$ at 0? What does this tell us about the limit of $x \sin(1/x)$?

4. We also looked at the important limit `Sin[x]/x`. As in (1), plot a graph of `Sin[x]/x`, along with the functions `1` and `Cos[x]`. (Again, for a bonus, color-code your graph).

5. Shrink the domain around $x = 0$ to see what happens. What are the limits of 1 and of $\cos(x)$ at 0?

Small Angle Approximation

1. `Plot[{Sin[x],x},{x,-1,1}]`

2. `Plot[{Sin[5x],5x},{x,-1,1}]`

3. `Plot[{Sin[x^2-4],x^2-4},{x,1,3}]`

4. `Plot[{Sin[Tan[x]],Tan[x]},{x,-1,1}]`

Filling in functions

We can always take a function whose domain is not all reals and extend it to a function whose domain is all real numbers by simply assigning values to all the points that aren't already covered. You can think of this as defining a piecewise function where one of the pieces is just a single point. We often want to extend a given function to be a *continuous* function on all reals; however, this is not always possible.

Go to the course webpage and download the file called "plot piecewise file". If the computer asks to open it with Mathematica, do that; if not, copy all the text (use ctrl+a) and then paste it into an empty Mathematica notebook, and when asked if you want to interpret the text, click "yes". Click anywhere in the giant block of code and hit shift+enter; this gives you the new function `PlotPiecewise` which we will be using for the rest of this lab.

For each of the following functions, do the following steps:

(a) Figure out the domain, and define the function `f[x]` in mathematica.

(b) Plot the function using the `PlotPiecewise` command from the notebook I posted on Moodle, and the command `PlotPiecewise[f[x],{x,-3,3}]`
   Use the domain given in the problem.

(c) Can you assign values to the function at the holes so that the function is "nice" or "connected" (continuous)? ("No" is a possible answer here, but think about why).

(d) If you can, define a new function `fFixed[x]` that has the holes filled in, using the `Piecewise` command. E.g. if there is a hole at $x = 2$ and you think the value "should" be 5, enter
   `fFixed[x_]:= Piecewise[{{f[x],x!= 2},{5,x==2}}]`
   (note we use `!=` for "not equals" and `==` for "equals")

(e) Plot your new function in `PlotPiecewise` to see if you have successfully filled the holes.
   `PlotPiecewise[fFixed[x],{x,-3,3}]`

1. `f[x_]:= (x^2-1)/(x-1)` on $(-2, 2)$.

2. `g[x_]:= 1/(x+1)` on $(-2, 2)$.

3. `h[x_]:= (x+1)^2/(x^2-1)` on $(-2, 2)$.

4. `i[x_]:= (x^2-2x-1)/(x-1)` on $(-2, 2)$.

5. `j[x_]:= 2 Abs[x+1]/(2x+2)` on $(-2, 2)$.

6. `k[x_]:= Sin[1/x]` on $(-1, 1)$.

7. `l[x_]:= x * Sin[1/x]` on $(-1, 1)$.

8. `m[x_]:= Sin[x-1]/(x-1)` on $(0, 2)$.

9. `n[x_]:= (x^3-8)/(x^2-4)` on $(-4, 4)$.

10. `o[x_]:= (x^2-x)/(x^3-3x^2+2x)` on$(-2, 4)$.