# Math 114 Spring 2017
# Calculus I Practice Homework 3.5 Solutions
# Do not turn in

1. From the definition, Prove that $\lim_{x\to+\infty} \frac{x+1}{x+3} = 1$.

   **Solution:** We guess the limit is 1. Let $\epsilon > 0$ and set $M = \underline{2/\epsilon \text{ or } 2/\epsilon - 3}$ . Suppose $x > M$. We can see that

   $$\left|\frac{x+1}{x+3} - 1\right| = \left|\frac{-2}{x+3}\right| = \frac{2}{|x+3|} = \frac{2}{x+3}$$

   since $x > M > 0$. Now there are two approaches we can take:

   (a)
   $$\frac{2}{x+3} < \frac{2}{M+3}$$
   so set $M = 2/\epsilon - 3$ and we have
   $$\frac{2}{x+3} < \frac{2}{M+3} = \frac{2}{2/\epsilon - 3 + 3} = \epsilon.$$

   (b) First observe
   $$\frac{2}{x+3} < \frac{2}{x} < \frac{2}{M}$$
   so set $M = 2/\epsilon$ and we have
   $$\frac{2}{x+3} < \frac{2}{x} < \frac{2}{M} = \frac{2}{2/\epsilon} = \epsilon.$$

2. From the definition, prove that $\lim_{x\to+\infty} \frac{x-1}{x^2} = 0$.

   **Solution:** We guess the limit is 0. Let $\epsilon > 0$ and set $M \geq 1, 1/\epsilon$. Then if $x > M$ we have

   $$\begin{aligned}\left|\frac{x-1}{x^2} - 0\right| &= \frac{|x-1|}{x^2} = \frac{x-1}{x^2} \leq \frac{x}{x^2} \\ &= \frac{1}{x} < \frac{1}{M} \leq \frac{1}{1/\epsilon} = \epsilon.\end{aligned}$$

3. ($\star$)From the definition, prove that $\lim_{x\to-\infty} \frac{x^2}{x+1} = -\infty$.

**Solution:** We guess $-\infty$. So let $N > 0$ and set $M = N$. Then if $x < -M$ we know that $x + 1 > x$ and thus $\frac{1}{x+1} < \frac{1}{x}$, so we have

$$\frac{x^2}{x+1} < \frac{x^2}{x} = x < -M = -N.$$