

Math 114 Spring 2017
Calculus I HW 1 Solutions
Due Friday, January 27

1. (a) Find two real numbers that solve $x^2 + 7x + 5 = 0$. **Solution:** By the quadratic formula, we have

$$x = \frac{-7 \pm \sqrt{49 - 20}}{2} = \frac{-7 \pm \sqrt{29}}{2}.$$

- (b) Factor $x^3 - 27$.

Solution: By the difference of cubes formula, we have $x^3 - 27 = (x^2 + 3x + 9)(x - 3)$. We can check our work by multiplying this out:

$$(x^2 + 3x + 9)(x - 3) = x^3 + 3x^3 + 9x - 3x^3 - 9x - 27 = x^3 - 27.$$

2. If $|f(x)| \leq |x|$ and $|g(x)| \leq 7 + x^2$, what can we say about $|f(x) + g(x)|$?

Solution: By the triangle inequality we know that

$$|f(x) + g(x)| \leq |f(x)| + |g(x)| \leq |x| + 7 + x^2.$$

By the reverse triangle inequality, we know that

$$\begin{aligned} |f(x) + g(x)| &\geq |f(x)| - |g(x)| \geq |f(x)| - 7 - x^2 \\ |g(x) + f(x)| &\geq |g(x)| - |f(x)| \geq |g(x)| - |x|. \end{aligned}$$

(We *cannot* replace the $|f(x)|$ with $|x|$ in the first inequality, because knowing that $|f(x)| \leq |x|$ cannot tell us anything about what $|f(x)|$ is *bigger* than. Similarly with $|g(x)|$ in the second inequality).

3. If $|f(x)| \geq 7$ and $|g(x)| \leq 3$, what can we say about $|f(x) + g(x)|$?

Solution: By the triangle inequality we know that

$$|f(x) + g(x)| \leq |f(x)| + |g(x)| \leq |f(x)| + 3.$$

Here we cannot do anything with the $|f(x)|$ term because we don't have an upper bound on it.

By the reverse triangle inequality, we know that

$$|f(x) + g(x)| \geq |f(x)| - |g(x)| \geq 7 - 3 = 4|g(x) + f(x)| \geq |g(x)| - |f(x)|.$$

This time we can replace $|f(x)|$ in the first inequality, since we know how *small* $f(x)$ is, not how big it is. We can't replace either $|g(x)|$ or $|f(x)|$ in the second inequality, so it's not really useful at all.

4. ★

(a) Find a pair of real numbers x and y such that $|x + y| < |x| + |y|$.

Solution: There are many correct solutions to all of these problems, but one example is $|1 + (-1)| < |1| + |-1|$.

(b) Find a pair of real numbers x and y such that $|x + y| = |x| + |y|$.

Solution: $|1 + 1| = |1| + |1|$.

(c) Find a pair of real numbers x and y such that $|x + y| > x + y$.

Solution: $|(-1) + (-1)| > (-1) + (-1)$.

5. ★

(a) Find a pair of real numbers x and y such that $|x + y| > |x| - |y|$.

Solution: $|1 + 1| > |1| - |1|$.

(b) Find a pair of real numbers x and y such that $|x + y| = |x| - |y|$.

Solution: $|1 + (-1)| = 1 - |-1|$.

(c) Find a pair of real numbers x and y such that $|x + y| < x - y$.

Solution: $|1 + (-1)| < 1 - (-1)$.