Math 114 Spring 2017
Calculus I HW 4 Solutions
Due Friday, February 17

1. Compute from the definition $\lim_{x\to 2^+} \frac{1}{x-2}$.

   Solution: Let $N > 0$ and set $\delta = \underline{1/N}$. Then if $2 < x < 2 < \delta$, we have $0 < x - 2 < \delta$ and

   $$\frac{1}{x-2} > \frac{1}{\delta} = \frac{1}{1/N} = N.$$

   Thus $\lim_{x\to 2^+} \frac{1}{x-2} = +\infty$.

2. Compute from the definition $\lim_{x\to 5} \frac{x-2}{x-5}$.

   Solution: Let $N > 0$ and set $\delta \leq \underline{2, 1/N}$. Then if $0 < |x-5| < \delta$, we have

   $$\begin{aligned} \left|\frac{x-2}{x-5}\right| &= \frac{|x-2|}{|x-5|} > \frac{|x-2|}{\delta} \\ &= \frac{|x-5+3|}{\delta} \geq \frac{3-|x-5|}{\delta} \\ &> \frac{3-\delta}{\delta} \geq \frac{1}{\delta} \geq \frac{1}{1/N} = N. \end{aligned}$$

   Thus $\lim_{x\to 5} \frac{x-2}{x-5} = \pm\infty$.

3. ($\star$)Compute from the definition $\lim_{x\to -3} \frac{-1}{(x+3)^4}$.

   Solution: Let $N > 0$ and set $\delta = \underline{1/\sqrt[4]{N}}$. Then if $|x+3| < \delta$, we have

   $$\begin{aligned} \frac{1}{(x+3)^4} &> \frac{1}{\delta^4} = \frac{1}{1/N} = N \\ \frac{-1}{(x+3)^4} &< \frac{-1}{\delta^4} = \frac{-1}{1/N} = -N. \end{aligned}$$

   Thus $\lim_{x\to -3} (x+3)^4 = -\infty$.

4. From the definition, compute $\lim_{x\to +\infty} \frac{x+1}{x+3}$.

Solution: We guess the limit is 1. Let $\epsilon > 0$ and set $M = \underline{2/\epsilon \text{ or } 2/\epsilon - 3}$. Suppose $x > M$. We can see that

$$\left|\frac{x+1}{x+3} - 1\right| = \left|\frac{-2}{x+3}\right| = \frac{2}{|x+3|} = \frac{2}{x+3}$$

since $x > M > 0$. Now there are two approaches we can take:

(a)
$$\frac{2}{x+3} < \frac{2}{M+3}$$

so set $M = 2/\epsilon - 3$ and we have

$$\frac{2}{x+3} < \frac{2}{M+3} = \frac{2}{2/\epsilon - 3 + 3} = \epsilon.$$

(b) First observe

$$\frac{2}{x+3} < \frac{2}{x} < \frac{2}{M}$$

so set $M = 2/\epsilon$ and we have

$$\frac{2}{x+3} < \frac{2}{x} < \frac{2}{M} = \frac{2}{2/\epsilon} = \epsilon.$$

5. From the definition, compute $\lim_{x\to+\infty} \frac{1+x}{x^2}$

Solution: We guess the limit is 0. Let $\epsilon > 0$ and set $M \geq 1, 2/\epsilon$. Then if $x > M$ we have

$$\begin{aligned}\left|\frac{1+x}{x^2} - 0\right| &= \frac{|1+x|}{x^2} \leq \frac{1+|x|}{x^2} \leq \frac{2x}{x^2} \\ &= \frac{2}{x} < \frac{2}{M} \leq \frac{2}{2/\epsilon} = \epsilon.\end{aligned}$$

6. ($\star$)From the definition, compute $\lim_{x\to-\infty} \frac{x^2}{x+1}$.

Solution: We guess $-\infty$. So let $N > 0$ and set $M = N$. Then if $x < -M$ we know that $x + 1 > x$ and thus $\frac{1}{x+1} < \frac{1}{x}$, so we have

$$\frac{x^2}{x+1} < \frac{x^2}{x} = x < -M = -N.$$

7. Explicitly naming the rule used in each step, calculate $\lim\limits_{x\to0} x^2 - 3x + 5$

Solution:

$$\begin{aligned}\lim_{x\to0} x^2 - 3x + 5 &= \lim_{x\to0} x^2 - \lim_{x\to0} 3x + \lim_{x\to0} 5 && \text{additivity}\\ &= \left(\lim_{x\to0} x\right)^2 - 3\lim_{x\to0} x + \lim_{x\to0} 5 && \text{exponents, scalars}\\ &= (0)^2 - 3\cdot 0 + 5 && \text{identity, constants}\\ &= 0 - 0 + 5 = 5.\end{aligned}$$

8. Explicitly naming the rule used in each step, calculate $\lim_{x\to 4} \sqrt{x} + \sqrt[3]{4+x}$

Solution:

$$\begin{aligned}
\lim_{x\to 4} \sqrt{x} + \sqrt[3]{4+x} &= \lim_{x\to 4} \sqrt{x} + \lim_{x\to 4} \sqrt[3]{4+x} && \text{additivity} \\
&= \sqrt{\lim_{x\to 4} x} + \sqrt[3]{\lim_{x\to 4} 4+x} && \text{exponents} \\
&= \sqrt{\lim_{x\to 4} x} + \sqrt[3]{\lim_{x\to 4} 4 + \lim_{x\to 4} x} && \text{additivity} \\
&= \sqrt{4} + \sqrt[3]{4+4} && \text{identity, constants} \\
&= 2 + 2 = 4.
\end{aligned}$$