Lab 2 Thursday February 2

Visualizing limits

Recall from last week that we can plot a function `f[x]`, on the domain $[a, b]$, with the command `Plot[f[x],{x,a,b}]` If we want to confine the output to the interval $[s, t]$ we can use the command `Plot[f[x],{x,a,b},PlotRange->{s,t}]`

Our goal for today is to represent limits graphically. Recall that for a limit $\lim_{x\to a} f(x) = L$ to exist, for any error margin ϵ we need to find a distance δ so that if x is within δ of a, then $f(x)$ is always within ϵ of L.

We'll start with two examples from class. First, we consider the function x^2.

1. Plot the function x^2 around the point $a = 0$ with the command `Plot[x^2, {x,-2,2}]` Guess/remember $\lim_{x\to 0} x^2$.

2. For now, let's set the error margin to $\epsilon = 1$. We can plot lines at $0 \pm \epsilon$ by running the command `Plot[{x^2,0-1,0+1},{x,-2,2}]` so that our error band is the area between the two lines. Alternatively, if we plot `Plot[x^2, {x,-2,2}, PlotRange->{-1,1}]` we can only see outputs that are within our error margin of $\epsilon = 1$.

   Based on this picture, if our input is between -2 and 2, will our output be within our error margin? What is the δ we are using for this picture—the horizontal distance we allow from zero—and is it close enough that our outputs are all inside the error margin?

3. What does δ need to be to make our output land in our error margin? Plot another graph with the same error margin but a smaller domain, so that all your outputs are within the error margin.

   (If you're using `PlotRange` and having trouble telling whether all your outputs are within the error margin, add the option `Filling->Automatic`).

4. If we use an error margin of $\epsilon = 1/4$, what δ do we need? Plot the corresponding graph.

5. Plot another graph for $\epsilon = 1/10$.

6. Come up with a formula for what δ needs to be in terms of ϵ. (hint: check your notes). Then use the following code:

   ```
   epsilon = 1
   delta = Sqrt[epsilon]
   Plot[x^2,{x,0-delta,0+delta},PlotRange->{0-epsilon,0+epsilon}
   ```

   Run this code with several different values of ϵ. Does it work every time?

7. (Optional) Run the code

   ```
   Table[Plot[x^2,{x,0-Sqrt[epsilon],0+Sqrt[epsilon]},Filling->Automatic,
   PlotRange->{0-epsilon,0+epsilon}],{epsilon,{1,1/2,1/4,1/10,1/100}}]
   ```

   to see graphs for a variety of different ϵ. (You can grab a copy of this worksheet from the course web page and copy and paste the code).

In the exercises, you will do the same thing for the limit as $x \to 3$, which we also did in class. I will also demonstrate for $f(x) = 1/x, a = 4$ and $f(x) = 1/x, a = 1$, which we discussed in class yesterday.

Exercises

Below there is a list of functions f paired with numbers a. For each item of the list:

1. Plot a graph of f centered at the point a.
2. Use this graph to estimate $L = \lim_{x \to a} f(x)$.
3. Plot a graph with an error margin given by $\epsilon = 1$. What δ do we need to make all outputs fall within ϵ of L?
4. Do the same with $\epsilon = 1/10$.
5. Find a formula for δ in terms of ϵ and test it for a few different ϵ.
6. (Optional) Make a table for graphs at different values of ϵ.

(a) $f(x) = x^2, a = 3$ (Check your notes!)

(b) $f(x) = 2x, a = -2$

(c) $f(x) = 1/x, a = 1$

(d) $f(x) = 1/x, a = 10$

(e) $f(x) = x^2 + 3, a = 0$

(f) $f(x) = \frac{x^2-4}{x-2}, a = 2$

(g) $f(x) = x^3 + x, a = 1$

(h) $f(x) = \frac{x-1}{x^2-1}, a = 1.$

Bonus: $f(x) = \sin(x), a = 0$