Lab 3 Thursday February 9

Piecewise Functions

We can define a piecewise function in Mathematica with the `Piecewise` command.

1. Define a piecewise function $f(x) = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$ with the command

   `f[x_] := Piecewise[{{-x^2, x<0}, {x^2, x>=0}}]`

   (notice that in Mathematica we use `>=` for $\geq$ and `<=` for $\leq$).

2. Look at the function and estimate the limit at 0. Then use the command `Limit[f[x],x->0]` to have Mathematica compute the limit. Then plot the function with domain $[-4, 4]$, with the command `Plot[f[x],{x,-4,4}]`.

3. Define a new function $g(x) = \begin{cases} -x^2 & x < -2 \\ x^2 & x > -2 \end{cases}$ and plot it. What is the limit at -2? Use the command `Limit[g[x],x ->-2`] to have Mathematica compute the limit. What happens? What do you think Mathematica is doing?

4. Come up with another piecewise function to test your theory, and have Mathematica compute the limit there.

5. Test the previous functions, but add the option `Direction->1`. For instance, run the command `Limit[g[x],x->-2,Direction->1]` What do you think this changes? Now try with `Direction->-1` instead. (Yes, this is backwards from how we'd like it).

6. Now plot f and g on one graph with domain $[-4, 4]$. What happens? The graph should look a little odd.

Bonus: Define the absolute value function as a piecewise function and plot it.

Plot the following functions, and error bounds that no delta will satisfy. Try putting in some error bounds that show the one-sided limits exist.:

1. $$H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

2. $$f(x) = \begin{cases} x & x < 1 \\ x + 2 & x > 1 \end{cases}$$

3. $$g(x) = \begin{cases} x^2 + x + 3 & x < -2 \\ x^5 - 1 & x > -2 \end{cases}$$

Infinite Limits

Look at the following functions and before graphing them guess:

1. At which points do you the limit to be infinite? In which directions?
2. What happens when the inputs get large?
3. Do you expect to find any zeroes?

Then plot the functions with the Mathematica `Plot` command. Remember to include a domain!

Coding tips:

- The horizontal asymptotes might be easier to see if the domain is large.
- You can download the "Plot Piecewise Code" from the course website to get a much better view of these graphs, using `PlotPiecewise` instead of `Plot`
- Remember you can use the `PlotRange` option with `Plot[f[x],{x,-5,5},PlotRange->{-15,15}]` (or with different numbers) to fix the height shown on the graph. This can be useful if too much information is hidden by the scale.
- Pay attention to parentheses! `1/x+1` is not the same thing as `1/(x+1)`.

(a) `1/(x^2-5x+6)`

(b) `1/(x^4+9x^3+29x^2+39x+18)`

(c) `(x-1)^(-2) (x-2)^(-2)`

(d) `(x-1)^(2) / (x-2)^2`

(e) `(x+1)/(Abs[x]-1)`

(f) `(x+1)/Abs[x -1]`
(Why do these two look so different?)

(g) `(2x^2 + 3x +1)/(Abs[x] * x + 1)`

(h) `Tan[x]`

(i) `x * Tan[x]`

(j) `Csc[x]`

(k) `x * Csc[x]`

(l) `x -x ^2`

(m) `1/(x - x^2)`

(n) `Sqrt[x^2+1]-x`