Lab 4 Thursday February 16

Exercises

1. Explicitly naming the rule used in each step, calculate $\lim_{x\to 2} \frac{x^2-4}{x-2}$.

   Solution:

$$\begin{aligned}
\lim_{x\to 2} \frac{x^2-4}{x-2} &= \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} \\
&= \lim_{x\to 2} x+2 && \text{Almost Identical Functions} \\
&= \lim_{x\to 2} x + \lim_{x\to 2} 2 && \text{additivity} \\
&= 2+2 && \text{identity and constants} \\
&= 4.
\end{aligned}$$

2. Explicitly naming the rule used in each step, calculate $\lim_{x\to 3} \frac{x^3-27}{x-3}$.

   Solution:

$$\begin{aligned}
\lim_{x\to 3} \frac{x^3-27}{x-3} &= \lim_{x\to 3} \frac{(x-3)(x^2+3x+9)}{x-3} \\
&= \lim_{x\to 3} x^2+3x+9 && \text{Almost Identical Functions} \\
&= \lim_{x\to 3} x^2 + \lim_{x\to 3} 3x + \lim_{x\to 3} 9 && \text{Additivity} \\
&= \left(\lim_{x\to 3} x\right)^2 + \lim_{x\to 3} 3x + \lim_{x\to 3} 9 && \text{Exponents} \\
&= \left(\lim_{x\to 3} x\right)^2 + 3\lim_{x\to 3} x + \lim_{x\to 3} 9 && \text{Scalars} \\
&= 3^2 + 3\cdot 3 + 9 = 27 && \text{Identity and constants}
\end{aligned}$$

3. Explicitly naming the rule used in each step, calculate $\lim_{x\to -1} \frac{x^2-2x-3}{x+1}$.

   Solution:

$$\begin{aligned}
\lim_{x\to -1} \frac{x^2-2x-3}{x+1} &= \lim_{x\to -1} \frac{(x+1)(x-3)}{x+1} \\
&= \lim_{x\to -1} x-3 && \text{Almost Identical Functions} \\
&= \lim_{x\to -1} x - \lim_{x\to -1} 3 && \text{Additivity} \\
&= (-1)-3 = -4 && \text{Identity and Constants}
\end{aligned}$$

4. Explicitly naming the rule used in each step, calculate $\lim_{x\to 4} (x+5)^{3/2}$

Solution:

$$\begin{aligned}
\lim_{x\to 4}(x+5)^{3/2} &= \left(\lim_{x\to 4} x+5\right)^{3/2} && \text{Exponents} \\
&= \left(\lim_{x\to 4} x + \lim_{x\to 4} 5\right)^{3/2} && = \text{Additivity} \\
&= (4+5)^{3/2} && \text{Identity and Constants} \\
&= 9^{3/2} = \left(\sqrt{9}\right)^3 = 3^3 = 27.
\end{aligned}$$

5. Using any techniques we have developed *in this course*, calculate $\lim_{x\to 5} \frac{x-5}{\sqrt{x+4}-3}$

Solution:

$$\begin{aligned}
\lim_{x\to 5} \frac{x-5}{\sqrt{x+4}-3} &= \lim_{x\to 5} \frac{x-5}{\sqrt{x+4}-3} \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3} \\
&= \lim_{x\to 5} \frac{(x-5)(\sqrt{x+4}+3)}{(x+4)-9} \\
&= \lim_{x\to 5} \frac{(x-5)(\sqrt{x+4}+3)}{x-5} \\
&= \lim_{x\to 5} \sqrt{x+4}+3 = \sqrt{9}+3 = 6.
\end{aligned}$$

6. Using any techniques we have developed *in this course*, calculate $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$

Solution:

$$\begin{aligned}
\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} &= \lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} \frac{\sqrt{x}+3}{\sqrt{x}+3} \\
&= \lim_{x\to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)} \\
&= \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}.
\end{aligned}$$

7. Using any techniques we have developed *in this course*, calculate $\lim_{x\to -2} \frac{\sqrt{x+6}-2}{x+2}$

Solution:

$$\begin{aligned}
\lim_{x\to -2} \frac{\sqrt{x+6}-2}{x+2} &= \lim_{x\to -2} \frac{\sqrt{x+6}-2}{x+2} \frac{\sqrt{x+6}+2}{\sqrt{x+6}+2} \\
&= \lim_{x\to -2} \frac{x+6-4}{(x+2)(\sqrt{x+6}+2)} \\
&= \lim_{x\to -2} \frac{1}{\sqrt{x+6}+2} = \frac{1}{2+2} = \frac{1}{4}
\end{aligned}$$