

Lab 5

Thursday March 2

The Squeeze Theorem

1. As a refresher, plot $x * \text{Sin}[1/x]$ on the same graph as $\text{Abs}[x]$ and $-\text{Abs}[x]$, with domain $[-1, 1]$ with the command

```
Plot[{x * Sin[1/x], Abs[x], -Abs[x]}, {x, -1, 1}]
```

Shrink the domain around the point $x = 0$ to see what happens. What does this tell us about the limit?

2. Try modifying this to use only the absolute value. Run the command

```
Plot[Abs[x * Sin[1/x]], {x, -1, 1}]
```

What functions bound this above and below? How does this compare to the previous plot?

3. We also discussed the important limit of $\text{Sin}[x]/x$, again using the squeeze theorem. As in (1), plot a graph of $\text{Sin}[x]/x$, along with 1 and $\text{Cos}[x]$. Shrink the domain around $x = 0$ to see what happens.

4. In class we discussed the function $f[x_] := (x-1)/(2+\text{Sin}[1/(x - 1)])$

- Plot the function f on the domain $[0, 2]$. Now plot it on the domain $].9, 1.9]$.
- Plot f together with the function $x - 1$. Is S always below that function? Is it always above it?
- Now plot s together with $\text{Abs}[x-1]$ and $\text{Abs}[x-1]/3$. Is the function trapped between these two functions?
- Try the pairs $\text{Abs}[x-1]$, $-\text{Abs}[x-1]/3$ and $\text{Abs}[x-1]$, $-\text{Abs}[x-1]$. What happens?

Bonus: Come up with a pair of functions that bounds f less tightly. Come up with a pair of functions that bounds it more tightly (hint: use piecewise functions).

5. Now do the same things, but with the function $\text{Abs}[f[x]]$. What does this look like? Why is it easier to work with?

6. • Plot a graph of the function $g[x_] := x^2/(1 + \text{Sin}[(x^2+12)/(x^3)]^2)$

(Hint: this is $\frac{x^2}{1 + \sin^2\left(\frac{x^2+12}{x^3}\right)}$). What is the limit?

- Try to find a function that bounds g from above. Then find a function that bounds g from below.
- Plot your three functions and see if g is truly squeezed between the other two.
- Repeat the process with $\text{Abs}[g[x]]$ How much does this change? Why?

Small Angle Approximation

1. `Plot[{Sin[x], x}, {x, -1, 1}]`
2. `Plot[{Sin[5x], 5x}, {x, -1, 1}]`
3. `Plot[{Sin[x^2-4], x^2-4}, {x, 1, 3}]`
4. `Plot[{Sin[Tan[x]], Tan[x]}, {x, -1, 1}]`

Filling in functions

We can always take a function whose domain is not all reals and extend it to a function whose domain is all real numbers by simply assigning values to all the points that aren't already covered. You can think of this as defining a piecewise function where one of the pieces is just a single point. We often want to extend a given function to be a *continuous* function on all reals; however, this is not always possible.

Go to the course webpage and download the file called “plot piecewise file”. If the computer asks to open it with Mathematica, do that; if not, copy all the text (use ctrl+a) and then paste it into an empty Mathematica notebook, and when asked if you want to interpret the text, click “yes”. Click anywhere in the giant block of code and hit shift+enter; this gives you the new function `PlotPiecewise` which we will be using for the rest of this lab.

For each of the following functions, do the following steps:

- (a) Figure out the domain, and define the function `f[x]` in mathematica.
- (b) Plot the function using the `PlotPiecewise` command from the notebook I posted on Moodle, and the command `PlotPiecewise[f[x], {x, -3, 3}]`
Use the domain given in the problem.
- (c) Can you assign values to the function at the holes so that the function is “nice” or “connected” (continuous)? (“No” is a possible answer here, but think about why).
- (d) If you can, define a new function `fFixed[x]` that has the holes filled in, using the `Piecewise` command. E.g. if there is a hole at $x = 2$ and you think the value “should” be 5, enter `fFixed[x_] := Piecewise[{{f[x], x != 2}, {5, x == 2}}`
(note we use `!=` for “not equals” and `==` for “equals”)
- (e) Plot your new function in `PlotPiecewise` to see if you have successfully filled the holes.
`PlotPiecewise[fFixed[x], {x, -3, 3}]`
 1. `f[x_] := (x^2-1)/(x-1)` on $(-2, 2)$.
 2. `g[x_] := 1/(x+1)` on $(-2, 2)$.
 3. `h[x_] := (x+1)^2/(x^2-1)` on $(-2, 2)$.
 4. `i[x_] := (x^2-2x-1)/(x-1)` on $(-2, 2)$.
 5. `j[x_] := 2 Abs[x+1]/(2x+2)` on $(-2, 2)$.
 6. `k[x_] := Sin[1/x]` on $(-1, 1)$.
 7. `l[x_] := x * Sin[1/x]` on $(-1, 1)$.
 8. `m[x_] := Sin[x-1]/(x-1)` on $(0, 2)$.
 9. `n[x_] := (x^3-8)/(x^2-4)` on $(-4, 4)$.
 10. `o[x_] := (x^2-x)/(x^3-3x^2+2x)` on $(-2, 4)$.