Lab 8 Thursday March 30

Hard to Solve Equations

Which of the following equations do you think you can solve for y? (That is, rewrite them with a solitary y on one side, and no ys on the other)?

For each equation, use the command `Solve[ y^3==x^2,y]` to have Mathematica solve the equation for y. (Note the double equals sign!) Notice that in at least one case you can probably do better than Mathematica can.

1. `y^3 +y == x^3 -x`
2. `x*y +x == 5`
3. `x^6 + CubeRoot[y] == 1`
4. `y^5 == y*x^2`
5. `y^5 + y^2 +1 ==0`
6. `Sin[x*y] == Cos[x * y]`
7. `y* Cos[x] == 1 + Sin[x*y]`
8. `Sqrt[x*y] == 1 + x^2 * y`

Implict Functions and their Tangents

When using the `ContourPlot` command, note the double `==` signs.

1. Yesterday in class, we showed that the tangent line to $x^3 + y^3 = 6xy$ at the point $(3, 3)$ is $y - 3 = 3 - x$. Verify this with the command
   `ContourPlot[{x^3 + y ^3 == 6 x*y, y-3 == 3 -x}, {x,-5,5},{y,-5,5}]`
2. (a) Use ContourPlot to plot the "cardioid" with equation:
   `x^2 + y^2 == ( 2x^2 + 2y^2 -x)^2`. (x and y domains from -1 to 1).
   (b) Compute the derivative at the point $(0, 1/2)$ by hand.
   (c) Check your computation by running the commands
   `D[x^2 + y[x]^2 == (2x^2 + 2y[x]^2 - x)^2,x]` and
   `D[x^2 + y[x]^2 == (2x^2 + 2y[x]^2 - x)^2,x] /. y[x] -> 1/2 /.x -> 0`
   Note some important details here. Mathematica can't figure out that `y` is a function of `x` instead of a constant unless we tell it, so we write `y[x]` instead of `y`. We can have mathematica automatically substitute for us, but it matters that we do `y[x]` before `x`. Why? Try it the other way and see what happens.
   (d) Plot the tangent line to the cardioid at that point in Mathematica.
   (e) What do you expect to happen if you try to find the tangent line at $(0, 0)$? Are you right? What does Mathematica say?
   (f) Looking at the graph, what do you think is the tangent line at the point $(1, 0)$? Can you get this from your derivative formula? Try computing the (implicit) derivative with respect to y instead of x. What happens?

3. (a) Plot the "devil's curve" `y^2(y^2 -4) == x^2 (x^2 - 5)`
   (b) Compute the derivative at $(0, -2)$ by hand.
   (c) Use Mathematica to check your answer.
   (d) Plot the devil's curve and its tangent line simultaneously.
   (e) (Just for fun) Run the command
   `ContourPlot[y^2(y^2-4) - x^2(x^2 -5),{x,-5,5},{y,-5,5}]` What happens? Why?

4. (a) Plot `(x^2 + y^2 -1)^3 - x^2 * y^3 == 0`
   (b) Check that $(1, 1)$ is a solution to this equation, and compute the derivative at $(1, 1)$. Use Mathematica to check your answer.
   (c) Plot the tangent line.
   (d) (Just for fun) Now try plotting without the equals sign, as in (3).

5. (a) Plot `Sin[x^2 + y^2] == Cos[x * y]` from -5 to 5.
   (b) Find the derivative at $(\sqrt{\pi/6}, \sqrt{\pi/6})$. (I know that looks terrible, but in context it's actually really easy to compute). Plot the tangent line.
   (c) (Just for fun) As before, replace the `==` with a `-` sign.

Bonus: Just for fun and pretty

1. Some other functions to try:
   - `Sin[Sin[x] + Cos[y]] == Cos[Sin[x * y] + Cos[y]]`
   - `Abs[Sin[x^2 - y^2]] == Sin[x + y] + Cos[x * y]`
   - `Csc[1-x^2] * Cot[2-y^2] == x * y`
   - `Abs[Sin[x^2 + 2 * x * y]] == Sin[x - 2 y]`
   - `(x^2 + y^2 - 3) Sqrt[x^2 + y^2] + .75 + Sin[4 Sqrt[x^2  + y^2]] Cos[84 ArcTan[y/x]] - Cos[6 ArcTan[y/x]] ==  0`

2. Try replacing the == signs with - signs.

3. Look at the examples on the Wolfram Alpha page
   `https://www.wolframalpha.com/examples/PopularCurves.html`

4. Search google for interesting pictures from implicit curves.