Problem 1. (a) Use the definition of limit to prove that $\lim_{x \to 2} \frac{1}{x+3} = \frac{1}{5}$.

(b) Use the definition of limit to prove that $\lim_{x \to 1} \frac{1}{(x-1)^2} = +\infty$.

Problem 2. (a) Use the Squeeze Theorem to show that $\lim_{x\to 5}(x-5)\sin\left(\frac{x^2+1}{x-5}\right)=0$.

(b) Compute $\displaystyle\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}$

(c) Compute $\displaystyle\lim_{x\to 5}\frac{e^{x-5}-1}{x-5}$

Problem 3. (a) **Directly from the definition**, compute $f'(1)$ where $f(x) = \sqrt{x+3}$.

(b) Compute $g'(x)$ where $g(x) = \ln\left|\dfrac{e^{\arctan(x^2)} - 5}{\sqrt[4]{x^2+1}}\right|$.

(c) Find a tangent line to the function $f(x) = \frac{e^x}{x}$ at the point given by $x = 2$.

Problem 4. (a) **Directly from the definition**, compute $f'(x)$ where $f(x) = \frac{1}{x-7}$.

(b) Write a tangent line to the curve $y^2 = x^{x\cos(x)}$ at the point $(\pi/2, -1)$.

(c) Find y' if $e^y + \ln(y) = x^2 + 1$.

Problem 5. (a) A cone with height h and base radius r has volume $\frac{1}{3}\pi r^2 h$. Suppose we have an inverted conical water tank, of height 4m and radius 6m. Water is leaking out of a small hole at the bottom of the tank. If the current water level is 2m and the water level is dropping at $\frac{1}{9\pi}$ meters per minute, what volume of water leaks out every minute?

(b) Use two iterations of Newton's method, starting at 0, to estimate the root of $e^x - 3x$.

(c) Let $g'(x) = g(x) + 3x$, and $g(2) = 4$. Use two steps of Euler's method to estimate $g(4)$. Is this an overestimate or an underestimate?

Problem 6. (a) A radioactive substance begins decaying from 100g of material. When it reaches 10g, it is decaying at rate of 1g per year. After how many years does this occur?

(b) Suppose that a company that produces and sells x units of a product makes a revenue of $R(x) = 100x - x^2/20$ and has costs given by $C(x) = 700 + 40x - x^2/100$. What is the maximum profit that can be made (where profit is revenues minus costs)?

(c) Ten miles from home you remember that you left the water running, which is costing you 90 cents an hour. Driving home at speed s miles per hour costs you $4(s/10)$ cents per mile. At what speed should you drive to minimize the total cost of gas and water?

Problem 7. (a) Find the absolute extrema of $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$ on $[0, 5]$.

(b) Find all the critical points of $f(x) = \dfrac{\ln(x)}{x^2 - 3x - 2}$

(c) Classify the relative extrema of $h(x) = \sqrt[3]{x}(x + 4)$

Problem 8. (a) Find all the critical points of $g(x) = \dfrac{x^2 - 8}{x + 3}$

(b) If $-1 \leq f'(x) \leq 3$ and $f(0) = 0$, what can you say about $f(4)$? Assume f is continuous and differentiable.

(c) Prove that $x^2 - (e^2 + 1)\ln(x)$ has exactly two real roots.

Problem 9. Sketch the graph of $j(x) = x^4 - 14x^2 + 24x + 6$. (Don't worry about finding roots).

Problem 10. Sketch a graph of the function $h(x) = e^x - \ln(|x|)$.