Math 114 Test 3

Instructor: Jay Daigle

- You will have 85 minutes to complete this exam.
- The exam has 5 "problems," each of which has multiple parts. Each problem is worth 30 points. The exam has 6 pages total.
- If you find a problem particularly difficult, skip it and come back. It may seem easier the second time, and even if it doesn't, you'll do better working on the other problems that seem easier.
- You may use one, one-sided sheet of handwritten notes.
- You may use a normal or scientific calculator. You may not use a graphing calculator. A calculator is not required to complete this exam.
- Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all. When in doubt, show more work and write complete sentences.
- If you need more paper to show work, I have extra at the front of the room.
- Good luck!

Name:

1	
2	
3	
4	
5	
$\sum$	

Problem 1. Compute the derivatives of the following functions using methods we have learned in class. Show enough work to justify your answers.

(a) $f(x) = \exp\left(\sec\left(\dfrac{\sqrt{x^2+1}+\log_5(x)}{x^4-\sin(x)}\right)\right)$

(b) $g(x) = \sin^4\left(\tan\left(\ln\left(x^2+1\right)\right)\right)$

Problem 2. (a) Find a formula for y' in terms of x and y if $xy^3 = \sqrt{x^2 + y^2}$.

(b) Compute $f'(5)$ where $f(x) = \log_3(\sqrt{x^2 + 2})$.

(c) Compute $g'(4)$ where $g(x) = 2^{\sqrt{x}}$.

Problem 3. (a) Let $j(x) = \sqrt{x^5 + 3x^3 + +5x}$. Find $(j^{-1})'(3)$.

(b) Find a tangent line to the curve given by $x^2y + (y - x)^2 = 5$ at the point $(2, 1)$.

Problem 4. (a) Use a tangent line approximation to estimate $\sqrt{7}$ using the derivative of $\sqrt{x}$ at the point 9.

(b) Suppose we have the differential equation $f'(t) = \frac{f(t)^2}{2} - t$, with $f(0) = 1$. Use Euler's method with three steps to approximate $f(3)$.

(c) Use two iterations of Newton's method, starting at -1 to find a root of $g(x) = x^3 + x^2 + 1$

Problem 5. (a) Spilled water is spreading in a circle whose area is increasing by 10π cm^2 per second. How fast is the radius increasing when the radius is 5 cm? (Recall the area of a circle is given by $A = \pi r^2$).

(b) A spectator is watching Usain Bolt run a 100m race. The spectator sits at the midpoint of the track (50m from each end), and 15 m away from the track. Usain Bolt runs 12m/s when he's at the 70m mark (yes, really). If the spectator is watching Bolt, how quickly is he rotating (how quickly is his angle from the track changing) when Bolt is at the 70m mark?