Math 214 Spring 2017
Linear Algebra HW 12
Due Friday, April 28

For all these problems, justify your answers.

1. Find the orthogonal decomposition of $(2, -1, 5, 6)$ with respect to $U = \text{Span}\{(1, 1, 1, 0), (1, 0, -1, 1)\}$.

2. Let V be a vector space and $L : V \to V$ a linear transformation, and let λ be a scalar. Prove that the eigenspace corresponding to λ is a subspace of V, using the subspace theorem.

3. Which of the following are eigenvectors of
$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}?$$
What are the corresponding eigenvalues?
$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

4. ($\star$) Let $V = \mathcal{D}(\mathbb{R}, \mathbb{R})$ be the space of differentiable real functions, and consider the linear transformation $\frac{d^2}{dx^2} : V \to V$. Find two linearly independent eigenvectors with eigenvalue 1. Find two linearly independent eigenvectors with eigenvalue -1.

5. Find all eigenvalues and the corresponding eigenvectors for
$$A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}.$$

6. Find the eigenvalues and corresponding eigenvectors for
$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 3 \\ 2 & 0 & 1 \end{bmatrix}.$$

7. Find the determinants of the following matrices. You should not need to perform any detailed computations for this problem.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & e & -2 \\ 2 & 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 2 & 1 & 3 \\ 0 & 4 & 1 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 1/4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 1 & 3 \\ -2 & 0 & -2 \\ 5 & 4 & 1 \end{bmatrix}.$$

8. Find the determinant of the following matrices:

$$A = \begin{bmatrix} 3 & 1 \\ 5 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 2 & 2 \\ -1 & 1 & 2 \\ 3 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} -4 & 1 & 3 \\ 2 & -2 & 4 \\ 1 & -1 & 0 \end{bmatrix}.$$

9. Suppose $A, B \in M_{n\times n}$ with $\det(A) = 3$ and $\det(B) = 5$. Find

   (a) $\det(A^{-1})$

   (b) $\det(AB^2)$

   (c) $\det(3B)$

   (d) $\det(B^T A)$.

10. Find the characteristic polynomial and the eigenvalues with multiplicity of the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 3 \\ 2 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$