Math 214 Spring 2017
Linear Algebra HW 2
Due Friday, February 3

1. ($\star$) Prove that $\mathcal{F}(\mathbb{R}, \mathbb{R})$, the set of functions from $\mathbb{R} \to \mathbb{R}$, is a vector space.

2. Prove that if $r\mathbf{u} = \mathbf{0}$, then either $r = 0$ or $\mathbf{u} = \mathbf{0}$.

3. ($\star$) Show that the zero vector is unique. That is, if $\mathbf{v}$ is a vector with the property that $\mathbf{v} + \mathbf{u} = \mathbf{u}$ for every vector $\mathbf{u} \in V$, then $\mathbf{v} = \mathbf{0}$.

4. (a) Show that the set $\{(x, x, y, y) | x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^4$.
   (b) Show that the set $\{(x, y, 0) | x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$. What does this subspace look like geometrically?
   (c) Show that the set $\{(x, 2x, 3x) | x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$. What does this subspace look like geometrically?

5. (a) Show that the set $\{f : \mathbb{R} \to \mathbb{R} | f(0) = 0\}$ is a vector space. (Hint: Show it is a subspace of something we know is a vector space).
   (b) Show that the set $\{f : \mathbb{R} \to \mathbb{R} | f(0) = 1\}$ is not a vector space.

6. (a) Show that if n is a positive integer, then the set $\mathcal{P}_n(x)$ of polynomials of degree at most n is a vector space.
   (b) Show that the set $\mathcal{C}(\mathbb{R}, \mathbb{R})$ the set of continuous functions of one real variable is a vector space.

7. Which of the following are vector spaces? You don't need to justify your answers.
   (a) $\{f : \mathbb{R} \to \mathbb{R} | f(0) = 0 \text{ and } f(1) = 0\}$
   (b) $\{f : \mathbb{R} \to \mathbb{R} | f(0) = 0 \text{ or } f(1) = 0\}$
   (c) $\{f : \mathbb{R} \to \mathbb{R} | f \text{ is constant}\}$
   (d) $\mathcal{C}([a, b], \mathbb{R})$ the space of continuous functions from $[a, b]$ to $\mathbb{R}$.

8. Which of the following are vector spaces? You don't need to justify your answers.
   (a) $\{(a, b) \in \mathbb{R}^2 | a + b = 0\}$
   (b) $\{(a, b) \in \mathbb{R}^2 | a + b = 3\}$
   (c) $\{a_0 + a_1x + a_2x^2 \in \mathcal{P}_2(x) | a_1 = 1\}$.
   (d) $\{a_0 + a_2x^2 + a_3x^3 + a_5x^5 \in \mathcal{P}_5(x)\}$.