Math 214 Spring 2017
Linear Algebra HW 9
Due Friday April 7

For all these problems, justify your answers.

1. Let $L(x, y, z) = (x - y, 3x + z, y - 2z)$. Find a formula for L^{-1}. (Do *not* leave your answer as a matrix).

2. Let $T : \mathbb{R}^3 \to \mathcal{P}_2(x)$ be given by $T(a, b, c) = (a - c) + (b - c)x + (a + b + c)x^2$. Find T^{-1}. (Do *not* leave your answer as a matrix).

3. ($\star$) If $f(x) \in \mathcal{P}_2(x)$ such that $f(1) = 4, f(3) = 7, f(4) = 1$, find $f(x)$. (Hint: define an evaluation map from $\mathcal{P}_2(x)$ to $\mathbb{R}^3$).

4. Let U, V be vector spaces, and $E = \{\mathbf{e}_1, \ldots, \mathbf{e}_n\}$ be a basis for U. Let $L : U \to V$ be a linear map. Prove that $L(E)$ spans $L(U)$.

5. Let E be the standard basis for $\mathbb{R}^3$, and let $F = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\}$.

   (a) Find the transition matrix corresponding to the change of basis from E to F.

   (b) For each of the following vectors (expressed in the standard basis), find the coordinates with respect to F: $(3, 2, 5)$; $(1, 1, 2)$; $(2, 3, 2)$.

6. Let $E = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\} = \left\{ \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$, and let $F = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$ be two bases for $\mathbb{R}^3$.

   (a) Find the transition matrix from E to F.

   (b) If $\mathbf{x} = 2\mathbf{e}_1 + 3\mathbf{e}_2 - 4\mathbf{e}_3$, find the coordinates of $\mathbf{x}$ with respect to F.

7. Let
$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x - y - z \\ -x + 2y - z \\ -x - y + 2z \end{bmatrix}.$$
   Let A be the matrix of L with respect to the standard basis, and let B be the matrix of L with respect to the basis $F = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$.

(a) Calculate B, the matrix of L with respect to F directly.

(b) Calculuate B by finding the matrix U corresponding to a change of basis from F to the standard basis, and calculating $U^{-1}AU$.

8. ($\star$) Let $T : \mathcal{P}_3(x) \to \mathcal{P}_3(x)$ be defined by $L(f(x)) = xf'(x) + f''(x)$.

   (a) Find the matrix A representing T with respect to $E = \{1, x, x^2\}$.

   (b) Find the matrix B representing T with respect to $F\{1, x, 1 + x^2\}$.

   (c) Find the matrix S such that $B = S^{-1}AS$.

   (d) If $p(x) = a_0 + a_1x + a_2(1 + x^2)$, calculate $T^n(p(x)) = T(T(\dots(T(p(x)))))$.