Math 214 Test 3
Practice Problems

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This is not a practice test, in the sense that it is not the format I expect the test to be. It is a collection of practice problems. I will update you when I finalize the test format.

I will post at least some solutions soon. I probably won't get a full solutions document written up, though we'll see what happens.

Proofs

1. Let Q be the subspace of $\mathcal{P}(x)$ consisting of polynomials with zero constant term. Prove that the function $D : Q \to \mathcal{P}(x)$ given by the derivative is an isomorphism.
2. Let $U = \operatorname{span}\{x, \sin(x), \cos(x), x^5, 1\}$. Find an isomorphism between U and $\mathbb{R}^5$.
3. Suppose V is a vector space and $L : V \to \mathbb{R}^5$ is surjective and $\dim \ker(L) = 2$. What can you say about V?
4. Suppose $T : \mathbb{R}^5 \to \mathcal{P}_4(x)$ and $\dim \ker(T) = 1$. What can you say about $T(\mathbb{R}^5)$?

Determine whether the following operators are invertible

1. $L(x, y, z) = (x, x + y, x + z)$
2. $L(x, y, z) = (x + y, y + z, x + z)$
3. $L(x, y, z, w) = (x + y, y + z, z + w, w + x)$
4. $L(x, y) = (x, y, 3x + 2y)$
5. $L(x, y, z) = (x + y, z + y)$
6. $L : \mathcal{P}_3(x) \to \mathbb{R}^3$ given by $L(f) = (f(1), f(2), f(3))$.

Find inverses for the following operators

1. $L(x, y) = (3x + y, 2x - 4y)$
2. $L(x, y, z) = (3x + y, 2y + 2z, x - z)$
3. $L(x, y, z) = (x + y + z, 2x - 2y, z)$
4. $L : \mathcal{P}_2(x) \to \mathbb{R}^3$ given by $L(f) = (f(0), f(1), f(2))$.

Find the transition matrices between the following bases

1. The standard basis and
$$F = \left\{ \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} \right\}$$

2. The standard basis and
$$F = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

3. $$E = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\} \quad \text{and} \quad F = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

4. $$E = \left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\} \quad \text{and} \quad F = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Write the given element in the given basis

1. Write $(3,1,4)$ in the basis $F = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

2. Write $(2,7,1)$ in the basis $F = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

3. Write $(1,-1,0)$ in the basis $F = \left\{ \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

4. Write $(2,3,4)$ in the basis $F = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$.

Find the matrix of the operator with respect to the given basis

1. Give the matrix of $L(x,y,z) = (3x+y+z, 5x-2y+z, y+z)$ with respect to $F = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$.

2. Give the matrix of $L(x,y,z) = (2x+3y-z, 4x-y+3z, 2x+z)$ with respect to $F = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$.

3. Give the matrix of $L(x,y,z) = (-x+4y+2z, 3x-5y+2, 3x+2y)$ with respect to $F = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$.

4. Give the matrix of $L(x,y,z) = (2x-y, 3x+y+4z, x+2y+z)$ with respect to $F = \left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$.

Angles and Magnitudes

1. Compute

$$\begin{bmatrix}3\\1\\2\end{bmatrix}\cdot\begin{bmatrix}5\\7\\-1\end{bmatrix},\quad \begin{bmatrix}4\\1\\3\\5\end{bmatrix}\cdot\begin{bmatrix}2\\-5\\7\\4\end{bmatrix},\quad \begin{bmatrix}1\\3\\2\end{bmatrix}\cdot\begin{bmatrix}4\\-1\\-1\end{bmatrix},\quad \begin{bmatrix}7\\1\\5\end{bmatrix}\cdot\begin{bmatrix}-3\\1\\1\end{bmatrix}.$$

2. Find the magnitudes and corresponding unit vectors for

$$\begin{bmatrix}3\\1\\2\end{bmatrix},\quad \begin{bmatrix}5\\12\end{bmatrix},\quad \begin{bmatrix}4\\2\\-2\end{bmatrix},\quad \begin{bmatrix}7\\-1\\-3\end{bmatrix}.$$

3. Find $\mathrm{proj}_{\mathbf{v}}\,\mathbf{u}$ for
   (a) $\mathbf{u} = (5,2), \mathbf{v} = (-3,4)$
   (b) $\mathbf{u} = (2,1), \mathbf{v} = (7,1)$
   (c) $\mathbf{u} = (3,1,4), \mathbf{v} = (2,1,1)$
   (d) $\mathbf{u} = (2,1,1), \mathbf{v} = (-4,-1,-1)$
   (e) $\mathbf{u} = (5,0,0), \mathbf{v} = (3,2,1)$.

Find parametric and normal equations for

1. $y = 5x + 2$
2. $2y = 3x + 4$
3. $y = 3x + 1, z = -2x + 3$
4. The line through $(3,4)$ and $(1,7)$
5. The line through $(0,1)$ and $(6,-3)$
6. $z = 3x + 2y - 2$
7. $z = -x + 4y + 3$
8. The plane through $(0,2,1), (5,2,1), (6,3,4)$
9. The plane through $(4,1,1), (-2,3,1), (5,-2,3)$.

Find the nearest point and the distance between

1. The point $(3,1)$ and the line $y = 4x + 2$
2. The point $(2,-2)$ and the line $y = 3x - 7$
3. The point $(2,1,4)$ and the line $z = 3x, y = 2x + 2$
4. The point $(2,4,4)$ and the plane $z = 3x + 2y + 1$
5. The point $(5,3,1)$ and the plane $z = 4x + 2$
6. The point $(2,2,2)$ and the plane through $(1,1,1), (3,4,5), (7,2,1)$.