Math 300: Junior Colloquium
Spring 2017
Discrete Math Topics List

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Your talk should have a clear statement of what you're talking about, an explanation of the topic, and at least one or two quick examples; it's especially nice to have an example where something doesn't work properly if that applies.

Do remember that you have at most five minutes or so, and also that your audience has in theory seen this material already; feel free to move through it quickly. Also make sure to check through the other topics to know what other people are covering, so you don't overlap or repeat too much.

I will want to have a copy of your notes/plans for the talk by 10:30 AM Wednesday–you can email to me a LaTeX file, or email me a scan/photo of your notes, or drop them off in the basket outside my office. If you're late on this, you will lose points. (Feel free to send it in on Tuesday or Monday). Once you enter your name on the Doodle poll you are responsible for that topic.

I will give you feedback Wednesday evening—probably a few pointers, and also making sure you have nothing outright false in your talk plans. Then on Thursday morning you will present your talk in the usual classroom.

1. Formal logic

   Define the logical operators (and, or, not, $\Rightarrow$) of formal logic, and the quantifiers (exists, for all).

2. Truth tables

   Define a predicate, and what it means for two logical statements to be equivalent. What is a truth table? Define a tautology and a contradiction

3. Sets and cardinality

   Explain "set-builder" notation. Define the power set of a set, and the cartesian product of two sets. Define the cardinality of a set. What is a countable set? What is an uncountable set? Brief reminder of how we prove a set is uncountable with diagonalization.

4. Union and intersection

   Define the union and the intersection of two sets. Define the complement of a set. State de Morgan's laws for union/intersection (and for and/or). Give some other set identities.

5. Induction

   Explain proof by induction. What is the difference between weak induction and strong induction?

6. Integers and divisibility

   Define integers and divisibility. What is a prime number? State the fundamental theorem of arithmetic.

7. GCD and the Euclidean algorithm

   Define the GCD of two numbers. Give the Euclidean algorithm for division-with-remainder and explain how we use it to compute the GCD of two numbers, with an example.

8. Modular arithmetic

   Define equivalence modulo n and modular arithmetic. What arithmetic/algebraic laws hold for modular arithmetic?

9. Relations

   Define a "relation" on a set. Define properties a relation can have: transitive, symmetric, reflexive, antisymmetric. Give examples and counterexamples for each property.

10. Equivalence classes

    What is an equivalence relation? What is an equivalence class on a set? Explain what it means that an equivalence class partitions a set.

11. Functions

    Define the domain, range, and pre-image of a function between two sets, with examples. Define the composition of two functions; when is it possible?

12. Inverses

    Define what it means for a function to be one-to-one, onto, or bijective. Give examples and counterexamples of each. What is the inverse of a function, and when does it have one?

13. Combinations and permutations

    What is the difference between a combination and a permutation? How do we find the numbers of combinations and permutations in a set? Give a couple examples. State the binomial theorem and how it relates combinatorially to combinations.

14. Principle of inclusion/exclusion

    Give the principle of inclusion and exclusion, and show how we can use it to count the elements of a set.