Math 300: Junior Colloquium
Spring 2017
Linear Algebra Topics List

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Your talk should have a clear statement of what you're talking about, an explanation of the topic, and at least one or two quick examples; it's especially nice to have an example where something doesn't work properly if that applies.

Do remember that you have at most five minutes or so, and also that your audience has in theory seen this material already; feel free to move through it quickly. Also make sure to check through the other topics to know what other people are covering, so you don't overlap or repeat too much.

I will want to have a copy of your notes/plans for the talk by 10:30 AM Wednesday–you can email to me a LaTeX file, or email me a scan/photo of your notes, or drop them off in the basket outside my office. If you're late on this, you will lose points. (Feel free to send it in on Tuesday or Monday). Once you enter your name on the Doodle poll you are responsible for that topic.

I will give you feedback Wednesday evening—probably a few pointers, and also making sure you have nothing outright false in your talk plans. Then on Thursday morning you will present your talk in the usual classroom.

1. Bases

   Define a linear combination. What does it mean for a set to be a spanning set? What does it mean to be linearly independent? Define a basis.

2. Matrices

   Define the matrix sum, scalar product, and matrix product operations. (Only this last needs to take much time or an example). Define the matrix transpose and give some properties, and define symmetric matrices.

3. Subspaces

   Define a subspace of $\mathbb{R}^n$, and the dimension of a subspace. Define the nullspace, rowspace, and columnspace of a matrix, and state the rank-nullity theorem.

4. Finding bases

   Talk about how to find bases for a subspace of $\mathbb{R}^n$, and how to find bases for the rowspace, columnspace, and nullspace of a matrix.

5. Invertible matrices

   Define the zero matrix and the identity matrix. Define the inverse of a matrix and what it means to be invertible. How can we tell if a matrix is singular (not invertible) or non-singular (invertible)?

6. Dot product

   Define the dot product on $\mathbb{R}^n$. Tell us how it gives the angle between two vectors, and the length/norm/magnitude of a vector. Discuss the triangle inequality and the Cauchy-Schwarz inequality.

7. Orthogonality

   Define what it means for two vectors to be orthogonal, and define an orthonormal basis. Give the Gram-Schmidt process for finding an orthonormal basis. Define the orthogonal complement of a space.

8. Geometry and projection

   Give the general form for the equation of a line and of a plane using vectors. Show how we can project onto a vector, and how we can project onto a subspace and into an orthogonal decomposition of a space.

9. Systems of linear equations

   Define a system of linear equations (versus non-linear), and what it means to be homogeneous or non–homogeneous. Define the coefficient matrix and the augmented matrix of a linear system. What does the columnspace of the matrix tell us about whether we can solve our system?

10. Gaussian elmination

    Define the elementary row operations and elementary matrices. Define row echelon form and reduced row echelon form. Explain how to use Gaussian elimination to solve a linear system, and Gauss-Jordan elimination to invert a matrix (or prove that it is not invertible).

11. Determinants

    Define the determinant and show how to compute it (with an example). Give some properties, and explain how row operations affect the determinant.

12. Eigenvectors

    Define the eigenvectors and eigenvalues of a matrix, and explain how to find them. Define the characteristic polynomial of a matrix, and explain the relationship between the eigenvalues and the determinant.

13. Similarity

    Define what it means for two matrices to be similar. How can we tell? Define a diagonal matrix, and then what it means for a matrix to be diagonalizable. When do we know a matrix is diagonalizable from its eigenvalues?