Math 300: Junior Colloquium
Spring 2017
Multivariable Calculus Topics List

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Your talk should have a clear statement of what you're talking about, an explanation of the topic, and at least one or two quick examples; it's especially nice to have an example where something doesn't work properly if that applies.

Do remember that you have at most five minutes or so, and also that your audience has in theory seen this material already; feel free to move through it quickly. Also make sure to check through the other topics to know what other people are covering, so you don't overlap or repeat too much.

I will want to have a copy of your notes/plans for the talk by 10:30 AM Wednesday–you can email to me a LaTeX file, or email me a scan/photo of your notes, or drop them off in the basket outside my office. If you're late on this, you will lose points. (Feel free to send it in on Tuesday or Monday). Once you enter your name on the Doodle poll you are responsible for that topic.

I will give you feedback Wednesday evening—probably a few pointers, and also making sure you have nothing outright false in your talk plans. Then on Thursday morning you will present your talk in the usual classroom.

1. Multivariable functions

   How do multivariable functions and three dimensional space work? How can we graph them. Sections and level surfaces.

2. Planes

   What do the equations for lines and planes look like? Different forms for equation of a plane, including normal form and form from three points. Tangent planes and using planes to approximate functions.

3. Vectors

   How do vectors work geometrically? Define the dot product, and explain how it relates to angles (including the formula with cosine). Define the cross product and give a geometric interpretation. How do we compute it?

4. Partial derivatives

   Define partial derivatives. Talk about higher–order partials and mixed partials—how do we compute them, and when do we know the different mixed partials are equal? Discuss the multivariable chain rules.

5. Gradients

   Define the gradient. How can we think about it geometrically, in terms of function increase and in terms of the level sets? What is a directional derivative and how do we compute it?

6. Optimization

   Optimization of multivariable functions. Define and identify critical points. How can we tell if they are maxima or minima?

7. Extreme Value Theorem

   State the extreme value theorem and explain what "closed" and "bounded" mean, and how we use the extreme value theorem in practice.

8. Lagrange multipliers

   Explain how to use Lagrange multipliers to do constrained optimization, and give some idea why. (Also explain what constrained optimization is). Explain how to optimize a function over the boundary of a region.

9. Multiple integrals

   Computing integrals of multivariable functions. Regions of integration, and choosing/changing orders of integration.

10. Parametrized curves

    Define parametrization and a parametrized curve. How do we parametrized lines and circles? Given a non-parametrized equation for a curve, how do we find a parametrization?

11. Vector fields and line integrals

    Define a vector field. How do we draw a vector field? Define/describe a line integral geometrically. Don't explain how to compute them, but do explain how we can predict the sign from a picture of the vector field and curve. Relate this to the physical idea of work.

12. Line integrals and computations

    Give the formula for a line integral over a vector field, and do a sample computation.

13. Conservative Vector Fields

    Define a conservative vector field. How do they relate to gradients? State path independence of line integrals for a conservative field. Explain how we can compute a line integral using the fundamental theorem in a conservative vector field. Observe that (and why) the integral over a closed loop in a conserative vector field is zero.

14. Green's theorem

    State Green's Theorem and show how to use it to compute integrals.