

Math 114 Spring 2018  
Calculus I HW 4 Solutions  
Due Wednesday, October 4

1. Let  $a$  and  $c$  be any constants. From the  $\epsilon$ - $\delta$  definition, prove that  $\lim_{x \rightarrow a} c = c$ .

**Solution:** Let  $\epsilon > 0$  and let  $\delta = 1$ . Then if  $0 < |x - a| < \delta$ , we compute

$$|f(x) - c| = |c - c| = 0 < \epsilon.$$

2. Stewart 1.4.22  
3. Stewart 1.4.24  
4. Stewart 1.4.26  
5. Stewart 1.6.14  
6. Stewart 1.6.18  
7. Stewart 1.6.20  
8. Stewart 1.6.22  
9. Stewart 1.6.24  
10. By any means we have developed in class, compute  $\lim_{x \rightarrow +\infty} x - \sqrt{x}$ .

**Solution:**

$$\begin{aligned} \lim_{x \rightarrow +\infty} x - \sqrt{x} &= \lim_{x \rightarrow +\infty} (x - \sqrt{x}) \cdot \frac{x + \sqrt{x}}{x + \sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{x^2 - x}{x + \sqrt{x}} \\ &= \lim_{x \rightarrow +\infty} \frac{1 - 1/x}{1/x + 1/x^{3/2}}. \end{aligned}$$

The top goes to 1 and the bottom goes to 0, so this limit is some form of infinity. Since  $x$  is approaching  $+\infty$ , the top and bottom are both always positive, so the limit is  $+\infty$ .