Math 114 Spring 2018
Calculus I HW 5 Solutions
Due Friday March 2

Make sure you have Stewart essential calculus early transcendentals second edition!

1. Stewart 1.4.34
2. Stewart 1.4.36
3. ($\star$) Using the squeeze theorem, show that

$$\lim_{x \to -2} \frac{x+2}{2 + \sin\left(\frac{1}{x+2}\right)} = 0.$$

Solution: We observe that

$$-1 \leq \sin\left(\frac{1}{x+2}\right) \leq 1$$

$$1 \leq 2 + \sin\left(\frac{1}{x+2}\right) \leq 3$$

$$1 \geq \frac{1}{2 + \sin\left(\frac{1}{x+2}\right)} \geq \frac{1}{3} \geq -1$$

$$|x+2| \geq \frac{x+2}{2 + \sin\left(\frac{1}{x+2}\right)} \geq -|x+2|$$

Then we compute $\lim_{x \to -2} |x+2| = 0$ and $\lim_{x \to -2} -|x+2| = 0$, so by the Squeeze Theorem,

$$\lim_{x \to -2} \frac{x+2}{2 + \sin\left(\frac{1}{x+2}\right)} = 0.$$

4. Stewart 1.4.50
5. Stewart 1.4.52 (Hint: what trig identities do we know? Can we make one of them show up?)
6. Stewart 1.4.54
7. ($\star$) Stewart 1.5.6

8. ($\star$) Stewart 1.5.8

9. Stewart 1.5.16

10. Let

$$f(x) = \begin{cases} x+3 & x > 2 \\ x^2+1 & x < 2 \end{cases}$$

Define a function that extends f and is continuous at all real numbers.

Solution: Define

$$f_F(x) = \begin{cases} x+3 & x > 2 \\ x^2+1 & x < 2 \\ 5 & x = 2 \end{cases}$$

Then f_F is continuous at 2 since $\lim_{x\to 2^-} f_F(x) = \lim_{x\to 2} x^2+1 = 5$ and $\lim_{x\to 2^+} f_F(x) = \lim_{x\to 2} x+3 = 5$.

11. Let

$$g(x) = \begin{cases} x^2-5 & x > -1 \\ 4x & x < -1 \end{cases}$$

Define a function that extends g and is continuous at all real numbers.

Solution: Define

$$g_F(x) = \begin{cases} x^2-5 & x > -1 \\ 4x & x < -1 \\ -4 & x = -1 \end{cases}$$

Then g_F is continuous at -1 since $\lim_{x\to -1^-} g_F(x) = \lim_{x\to -1} 4x = -4$ and $\lim_{x\to -1^+} g_F(x) = \lim_{x\to -1} x^2-5 = -4$.

12. Stewart 1.5.30