Math 114 Spring 2017
Calculus I HW 7 Solutions
Due Friday, March 30

1. Stewart 2.5.36
2. Stewart 2.5.42
3. Find

$$\frac{d}{dx}\sqrt[5]{\frac{x^2\sin(3x)}{\tan(x)}}$$

Solution:

$$\frac{1}{5}\left(\frac{x^2\sin(3x)}{\tan(x)}\right)^{-4/5}\frac{(2x\sin(3x)+x^2\cos(3x)3)\tan(x)-\sec^2(x)x^2\sin(3x)}{\tan^2(x)}$$

4. Find

$$\frac{d}{dx}\tan^4(\sqrt[3]{x^5+x^3+2}+1).$$

Solution:

$$\begin{aligned}\frac{d}{dx}\tan^4(\sqrt[3]{x^5+x^3+2}+1) &= 4\tan^3(\sqrt[3]{x^5+x^3+2}+1)\cdot\sec^2(\sqrt[3]{x^5+x^3+2}+1)\\ &\quad\cdot(\sqrt[3]{x^5+x^3+2}+1)'\\ &= 4\tan^4(\sqrt[3]{x^5+x^3+2}+1)\sec^2(\sqrt[3]{x^5+x^3+2}+1)\\ &\quad\cdot\left(\frac{1}{3}(x^5+x^3+1)^{-2/3}\cdot(5x^4+3x^2)\right).\end{aligned}$$

5. Stewart 2.8.5 (no graphing)
6. Stewart 2.8.12
7. ($\star$) Stewart 2.8.16
8. Stewart 2.6.10

9. Stewart 2.6.20

10. Stewart 2.6.22

11. Stewart 2.6.26

12. Suppose $f(x) = ax^2 + bx + c$ satisfies $f(2) = 1, f'(2) = 2, f''(2) = 3$. Find $f(x)$.

    Solution: We have $1 = f(2) = a(2)^2 + b(2) + c = 4a + 2b + c$; we have $2 = f'(2) = 2a(2) + b = 4a + b$; and we have $3 = f''(2) = 2a$. This tells us that $a = 3/2$. Thus $2 = 6+b$ so $b = -4$. Finally, we get $1 = 6-8+c$ so $c = 3$. Thus $f(x) = 3x^2/2-4x+3$.

13. Prove that $f(x) = \sin(x) + x^2$ satisfies $f''(x) + f(x) = x^2 + 2$.

    Solution: We see that $f'(x) = \cos(x) + 2x$ and $f''(x) = -\sin(x) + 2$. Thus

    $$f''(x) + f(x) = -\sin(x) + 2 + \sin(x) + x^2 = x^2 + 2$$

    as desired.