Lab 2 Tuesday February 1

Visualizing limits

Recall from last week that we can plot a function `f[x]`, on the domain $[a, b]$, with the command `Plot[f[x],{x,a,b}]`

Our goal for today is to represent limits graphically. Recall that for a limit $\lim_{x\to a} f(x) = L$ to exist, for any error margin ϵ we need to find a distance δ so that if x is within δ of a, then $f(x)$ is always within ϵ of L.

We'll start with an example. Let's consider the function x^2.

1. Plot the function x^2 around the point $a = 0$ with the command `Plot[x^2, {x,-2,2}]` Guess/remember $\lim_{x\to 0} x^2$.

2. For now, let's set the error margin to $\epsilon = 1$. We can plot lines at $0 \pm \epsilon$ by running the command `Plot[{x^2,0-1,0+1},{x,-2,2}]` so that our error band is the area between the two lines. Based on this picture, if our input is between -2 and 2, will our output be within our error margin? What is the δ we are using for this picture—the horizontal distance we allow from zero—and is it close enough that our outputs are all inside the error margin?

3. What does δ need to be to make our output land in our error margin? Plot another graph with the same error margin but a smaller domain, so that all your outputs are within the error margin.

4. If we use an error margin of $\epsilon = 1/4$, what δ do we need? Plot the corresponding graph.

5. Plot another graph for $\epsilon = 1/10$.

Bonus: Come up with a formula for what δ needs to be, in terms of ϵ. We'll discuss this in detail in tomorrow's class. Then use the following code:

```
epsilon = 1

delta = Sqrt[epsilon]

Plot[x^2,{x,0-delta,0+delta},PlotRange->{0-epsilon,0+epsilon}]
```

Run this code with several different values of ϵ. Does it work every time?

I will also demonstrate for $f(x) = 1/x, a = 4$ and $f(x) = 1/x, a = 1$. In the exercises you will do this same process with a number of functions.

Extra Visualization: Two Deltas

In class we looked at the limit of $f(x) = x^2$ as x approached 3. We wound up taking $\delta \leq 1, \epsilon/7$. Why did we have both of those limits?

Recall that δ is like $|x - 3|$. We computed in class that if $\delta \leq 1$, then $|x^2 - 9| < 7|x - 3| < 7\delta$. We can plot these two functions with `Plot[{Abs[x^2-9],7Abs[x-3]},{x,2,4}]` and see the bound in the graph. But if we expand the domain to `{x,1,5}` then our bound no longer works. We'd need to change our slope from 7 to 8.

We can plot this same observation with the actual function. Now we'll plot $f(x)$ and $L + |x - 3|$, and get `Plot[{x^2,9+7Abs[x-3]},{x,2,4}]` Again, we can see the bound works; but if we expand the window so that $\delta > 1$ then the bound fails.

Exercises

Below there is a list of functions f paired with numbers a. For each item of the list:

1. Plot a graph of f centered at the point a.
2. Use this graph to estimate $L = \lim_{x \to a} f(x)$.
3. Plot a graph with an error margin given by $\epsilon = 1$. What δ do we need to make all outputs fall within ϵ of L?
4. Do the same with $\epsilon = 1/2, 1/10, 1/100$.

(a) $f(x) = 2x, a = -2$

(b) $f(x) = 1/x, a = 1$

(c) $f(x) = 1/x, a = 10$

(d) $f(x) = 3, a = 0$

(e) `f[x_]:=Abs[x]/x`

(f) $f(x) = x^2 + 3, a = 0$

(g) $f(x) = \frac{x^2-4}{x-2}, a = 2$

(h) $f(x) = x^3 + x, a = 1$

(i) $f(x) = \frac{x-1}{x^2-1}, a = 1.$

Bonus: $f(x) = \sin(x), a = 0$