Lab 3　　Thursday February 8

Piecewise Functions

First, go to the course website, and download the "Plot Piecewise file". You should get a file called `PlotPiecewise.nb`. Open this in Mathematica. You will get a notebook with a big pile of code. You don't need to read the code, but click anywhere inside the code, and hit shift-enter to evaluate. This will give us a `PlotPiecewise` command to replace our usual `Plot` command.

We can define a piecewise function in Mathematica with the `Piecewise` command.

1. Define a piecewise function $f(x) = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$ with the command

   ```
   f[x_] := Piecewise[{{-x^2, x<0}, {x^2, x>=0}}]
   ```

   (notice that in Mathematica we use `>=` for $\geq$ and `<=` for $\leq$).

2. Look at the function and estimate the limit at 0. Then use the command `Limit[f[x],x->0]` to have Mathematica compute the limit. Then plot the function with domain $[-4, 4]$, with the command `PlotPiecewise[f[x],{x,-4,4}]`.

3. Define a new function $g(x) = \begin{cases} -x^2 & x < -2 \\ x^2 & x > -2 \end{cases}$ and plot it with the `PlotPiecewise` command. What is the limit at -2?

   Use the command `Limit[g[x],x ->-2]` to have Mathematica compute the limit. What happens? What do you think Mathematica is doing?

4. Come up with another piecewise function to test your theory, and have Mathematica compute the limit there.

5. Test the previous functions, but add the option `Direction->1`. For instance, run the command `Limit[g[x],x->-2,Direction->1]` What do you think this changes? Now try with `Direction->-1` instead. (Yes, this is backwards from how we'd like it).

6. Now plot f and g on one graph with domain $[-4, 4]$. What happens? The graph should look a little odd.

Bonus: Define the absolute value function as a piecewise function and plot it.

Plot each of the following functions. Can you find a point where it looks like no limit exists? Try to plot a pair of horizontal lines that the function never stays between.

1. $$H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

2. $$f(x) = \begin{cases} x & x < 1 \\ x + 2 & x \geq 1 \end{cases}$$

3. $$g(x) = \begin{cases} x^2 + x + 3 & x < -2 \\ x^5 - 1 & x \geq -2 \end{cases}$$

4. $$h(x) = \begin{cases} x - 1 & x < -1 \\ 4 - 2x & x \geq -2 \end{cases}$$

Infinite Limits

Look at the following functions and, before graphing them, guess where the vertical asymptotes are. Then plot the functions with the Mathematica `Plot` command. Remember to include a domain! `{x,-5,5}` will work for most of these, but you might need to play around a bit. Some of the graphs will look better if you add the `PlotRange->{-10,10}` option, as with `Plot[f[x],{x,-5,5},PlotRange->{-10,10}]` for an example.

Important note: make sure ot pay attention to parentheses! `1/x+1` is not the same thing as `1/(x+1)`.

(a) `1/(x^2-5x+6)` (`PlotRange->{-10,10}`)

(b) `1/(x^4+9x^3+29x^2+39x+18)` (same)

(c) `(x-1)^(-2) (x-2)^(-2)`

(d) `(x-1)^(2) / (x-2)^2`

(e) `(x+1)/(Abs[x]-1)`

(f) `(x+1)/Abs[x -1]`
(Why do these two look so different?)

(g) `(2x^2 + 3x +1)/(Abs[x] * x + 1)`

(h) `Tan[x]`

(i) `x * Tan[x]`

(j) `Csc[x]`

(k) `x * Csc[x]`

(l) `x -x ^2`

(m) `1/(x - x^2)`

(n) `Sqrt[x^2+1]-x`