

Math 114 Spring 2018  
Calculus I Practice Homework 3.5 Solutions  
Do not turn in

1. Prove that  $\lim_{x \rightarrow 5} \frac{x-2}{x-5} = \pm\infty$ .

**Solution:** Let  $N > 0$  and set  $\delta \leq \underline{2, 1/N}$ . Then if  $0 < |x - 5| < \delta$ , we have

$$\begin{aligned} \left| \frac{x-2}{x-5} \right| &= \frac{|x-2|}{|x-5|} > \frac{|x-2|}{\delta} \\ &= \frac{|x-5+3|}{\delta} \geq \frac{3-|x-5|}{\delta} \\ &> \frac{3-\delta}{\delta} \geq \frac{1}{\delta} \geq \frac{1}{1/N} = N. \end{aligned}$$

Thus  $\lim_{x \rightarrow 5} \frac{x-2}{x-5} = \pm\infty$ .

2. Prove that  $\lim_{x \rightarrow -3} \frac{-1}{(x+3)^4} = -\infty$ .

**Solution:** Let  $N > 0$  and set  $\delta = \underline{1/\sqrt[4]{N}}$ . Then if  $|x + 3| < \delta$ , we have

$$\begin{aligned} \frac{1}{(x+3)^4} &> \frac{1}{\delta^4} = \frac{1}{1/N} = N \\ \frac{-1}{(x+3)^4} &< \frac{-1}{\delta^4} = \frac{-1}{1/N} = -N. \end{aligned}$$

Thus  $\lim_{x \rightarrow -3} (x+3)^4 = -\infty$ .

3. Prove that  $\lim_{x \rightarrow 4} \frac{3}{(x-4)^2} = +\infty$ .

**Solution:** Let  $N > 0$  and set  $\delta \leq \underline{\sqrt{3/N}}$ . If  $0 < |x - 4| < \delta$ , then

$$\left| \frac{3}{(x-4)^2} \right| = \frac{3}{|x-4|^2} > \frac{3}{\delta^2} \geq \frac{3}{(\sqrt{3/N})^2} = N.$$

4. Explicitly naming the rule used in each step, calculate  $\lim_{x \rightarrow 0} x^2 - 3x + 5$

**Solution:**

$$\begin{aligned}\lim_{x \rightarrow 0} x^2 - 3x + 5 &= \lim_{x \rightarrow 0} x^2 - \lim_{x \rightarrow 0} 3x + \lim_{x \rightarrow 0} 5 && \text{additivity} \\ &= \left(\lim_{x \rightarrow 0} x\right)^2 - 3 \lim_{x \rightarrow 0} x + \lim_{x \rightarrow 0} 5 && \text{exponents, scalars} \\ &= (0)^2 - 3 \cdot 0 + 5 && \text{identity, constants} \\ &= 0 - 0 + 5 = 5.\end{aligned}$$

5. Explicitly naming the rule used in each step, calculate  $\lim_{x \rightarrow 4} \sqrt{x} + \sqrt[3]{4+x}$

**Solution:**

$$\begin{aligned}\lim_{x \rightarrow 4} \sqrt{x} + \sqrt[3]{4+x} &= \lim_{x \rightarrow 4} \sqrt{x} + \lim_{x \rightarrow 4} \sqrt[3]{4+x} && \text{additivity} \\ &= \sqrt{\lim_{x \rightarrow 4} x} + \sqrt[3]{\lim_{x \rightarrow 4} 4+x} && \text{exponents} \\ &= \sqrt{\lim_{x \rightarrow 4} x} + \sqrt[3]{\lim_{x \rightarrow 4} 4 + \lim_{x \rightarrow 4} x} && \text{additivity} \\ &= \sqrt{4} + \sqrt[3]{4+4} && \text{identity, constants} \\ &= 2 + 2 = 4.\end{aligned}$$

6. Explicitly naming the rule used in each step, calculate  $\lim_{x \rightarrow 2} f(x)$  where

$$f(x) = \begin{cases} x + 1 & x < 2 \\ x^2 - 1 & x > 2 \end{cases}$$

**Solution:**

We need to calculate the two one-sided limits. We have

$$\begin{aligned}\lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} x + 1 && \text{Almost Identical Functions} \\ &= \lim_{x \rightarrow 2^-} x + \lim_{x \rightarrow 2^-} 1 && \text{Additivity} \\ &= \lim_{x \rightarrow 2^-} x + 1 && \text{Constants} \\ &= 2 + 1 = 3 && \text{Identity} \\ \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} x^2 - 1 && \text{Almost Identical Functions} \\ &= \lim_{x \rightarrow 2^+} x^2 - \lim_{x \rightarrow 2^+} 1 && \text{Additivity} \\ &= \left(\lim_{x \rightarrow 2^+} x\right)^2 - \lim_{x \rightarrow 2^+} 1 && \text{Exponents} \\ &= (2)^2 - \lim_{x \rightarrow 2^+} 1 && \text{Identity} \\ &= 4 - 1 = 3 && \text{Constants}\end{aligned}$$

We see that  $\lim_{x \rightarrow 2^+} f(x) = 3 = \lim_{x \rightarrow 2^-} f(x)$ , so by agreement of the left and right limits,  $\lim_{x \rightarrow 2} f(x) = 3$ .