Math 212 Spring 2018
Multivariable Calculus Practice HW 7.5 Solutions
For Test 3 on Wednesday, April 11

1. 17.3.27
2. 17.3.29
3. 17.3.37
4. 17.4.1
5. 17.4.5
6. 17.4.7
7. 17.4.16
8. 18.1.1
9. 18.1.3
10. 18.1.5
11. 18.1.7 (without computing an integral)
12. 18.1.13 (without computing an integral)
13. 18.1.25
14. 18.2.1
15. 18.2.3
16. 18.2.5
17. 18.2.9
18. 18.2.11
19. 18.2.17
20. 18.2.19
21. 18.2.21
22. 18.2.25
23. 18.2.29
24. 18.2.31

1. Find the mass of a wire lying along the straight line from $(1,1)$ to $(2,4)$ with density $3x+2y$.

   Solution: Write $\vec{r}(t) = (1+t, 1+3t)$ for $0 \le t \le 1$. Then

$$\int_C 3x + 2y \, d\vec{r} = \int_0^1 (3+3t+2+6t)\sqrt{(1+9)}\, dt = \int_0^1 5\sqrt{10} + 9t\sqrt{10}\, dt$$
$$= 5\sqrt{10}t + 9t^2\sqrt{10}/2|_0^1 = 5\sqrt{10} + 9\sqrt{10}/2 = 19\sqrt{10}/2.$$

2. Find the mass of a wire lying along the arc of the unit circle from $(0,1)$ to $(1,0)$ with density xy.

   Solution: We can parametrize $\vec{r}(t) = (\cos(t), \sin(t))$ for $0 \le t \le \pi/2$ (recall that orientation doesn't matter). Then

$$\int_C xy\, d\vec{r} = \int_0^{\pi/2} \cos(t)\sin(t)\sqrt{\cos^2 t + \sin^2 t}\, dt$$
$$= \sin^2(t)/2|_0^{\pi/2} = 1/2.$$

3. Consider a wire lying along the pathparametrized by $\vec{r}(t) = (3t^2 - 2, t^2 + 1)$ for $1 \leq t \leq \sqrt{2}$. If the density is given by $\delta(x, y) = x + y$, calculate the mass of the wire.

   Now sketch this parametrization. What easier thing could you have done?

   Solution:

$$\int_C \delta\, d\vec{r} = \int_1^{\sqrt{2}} ((3t^2 - 2) + (t^2 + 1))\|(6t, 2t)\|\, dt$$
$$= \int_1^{\sqrt{2}} (4t^2 - 1)2t\sqrt{10}\, dt$$
$$= \int_1^{\sqrt{2}} 8\sqrt{10}t^3 - 2\sqrt{10}t\, dt$$
$$= 2\sqrt{10}t^4 - \sqrt{10}t^2|_1^{\sqrt{2}} = 8\sqrt{10} - 2\sqrt{10} - (2\sqrt{10} - \sqrt{10}) = 5\sqrt{10}.$$

   We might notice that $\vec{r}'(t) = (6t, 2t)$ has the same direction for all t, just a different speed—which means that it's parametrizing a line. Plugging in $t = 1, \sqrt{2}$ says this is the line from $(1, 2)$ to $(4, 3)$, which we can parametrize with $\vec{r}(t) = (3t + 1, t + 2)$ for $0 \leq t \leq 1$.

4. Compute $\int_C f\, d\vec{r}$ if $\vec{r}(t) = (2t, t^3/3, t^2)$ for $0 \leq t \leq 2$ and $f(x, y, z) = 3yz + x$.

   Solution:

$$\int_C f\, d\vec{r} = \int_0^2 (2t^5 + 2t)\sqrt{4 + t^4 + 4t^2}\, dt$$
$$= \int_0^2 (2t^5 + 2t)(t^2 + 2)\, dt = \int_0^2 2t^7 + 4t^5 + 2t^3 + 4t\, dt$$
$$= t^8/4 + 2t^6/3 + t^4/2 + 2t^2|_0^2 = 64 + 128/3 + 8 + 8 = 368/3.$$