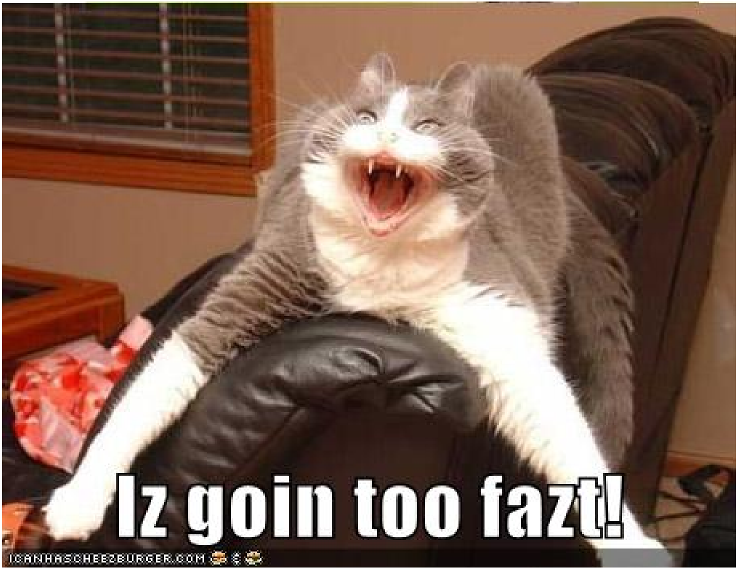
Towards a Classification of 3×3 C-Symmetric Matrices

Jay Daigle
Advisor: Stephan Garcia
gjd02004@mymail.pomona.edu
http://www.dci.pomona.edu/~jadagul

Pomona College

February 22, 2008


i can has
attenshun plz?


Iz goin too fazt!
ICANHASCHEEZBURGER.COM

Motivating Examples

Motivating Examples

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

Motivating Examples

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

Motivating Examples

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} \frac{1+\sqrt{3}i}{2} & 0 & 0 \\ 0 & \frac{1-\sqrt{3}i}{2} & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Motivating Examples

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} \frac{1+\sqrt{3}i}{2} & 0 & 0 \\ 0 & \frac{1-\sqrt{3}i}{2} & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -5 & 0 & 4 \\ -4 & -2 & 2 \\ 1 & 4 & -3 \end{pmatrix}$$

Motivating Examples

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} \frac{1+\sqrt{3}i}{2} & 0 & 0 \\ 0 & \frac{1-\sqrt{3}i}{2} & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -5 & 0 & 4 \\ -4 & -2 & 2 \\ 1 & 4 & -3 \end{pmatrix}$$

Motivating Examples

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} \frac{1+\sqrt{3}i}{2} & 0 & 0 \\ 0 & \frac{1-\sqrt{3}i}{2} & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -5 & 0 & 4 \\ -4 & -2 & 2 \\ 1 & 4 & -3 \end{pmatrix}$$

Fact

Every matrix is similar to a complex symmetric matrix.

Unitary Equivalence

Unitary Equivalence

Definition

Let U be a $n \times n$ matrix. If U is an invertible isometry, then we say U is unitary.

Unitary Equivalence

Definition

Let U be a $n \times n$ matrix. If U is an invertible isometry, then we say U is unitary.

Definition

If A and B are $n \times n$ matrices and $A = UBU^{-1}$ for some unitary matrix U, then A is unitarily equivalent to B.

Outlines of the Problem

Outlines of the Problem

- Every square matrix is similar to a complex symmetric matrix (CSM).

Outlines of the Problem

- Every square matrix is similar to a complex symmetric matrix (CSM).
- Not every square matrix is unitarily equivalent to a CSM (UECSM).

Outlines of the Problem

- Every square matrix is similar to a complex symmetric matrix (CSM).
- Not every square matrix is unitarily equivalent to a CSM (UECSM).
- Develop techniques to tell the difference.

Outlines of the Problem

- Every square matrix is similar to a complex symmetric matrix (CSM).
- Not every square matrix is unitarily equivalent to a CSM (UECSM).
- Develop techniques to tell the difference.
- Classify 3×3 UECSM.

Why is this hard?

Why is this hard?

Most useful invariants are similarity invariants.

Why is this hard?

Most useful invariants are similarity invariants.

$$\begin{aligned}
\det(A) &= \det(Q^{-1}BQ) \\
&= \det(Q^{-1})\det(B)\det(Q) \\
&= \det(Q^{-1})\det(Q)\det(B) \\
&= \det(I)\det(B) = \det(B)
\end{aligned}$$

Why is this hard?

Most useful invariants are similarity invariants.

$$\begin{aligned}
\det(A) &= \det(Q^{-1}BQ) \\
&= \det(Q^{-1})\det(B)\det(Q) \\
&= \det(Q^{-1})\det(Q)\det(B) \\
&= \det(I)\det(B) = \det(B)
\end{aligned}$$

- Determinant
- Trace
- Eigenvalues
- Rank
- Minimum Polynomial
- Jordan Form

The Conjugate Transpose T^*

The Conjugate Transpose T^*

Definition

If $T = (a_{ij})$ is a square complex matrix, then we say that $T^* = (\overline{a_{ji}})$ is its conjugate transpose.

The Conjugate Transpose T^*

Definition

If $T = (a_{ij})$ is a square complex matrix, then we say that $T^* = (\overline{a_{ji}})$ is its conjugate transpose.

$$T = \begin{pmatrix} 1 & 6 & i \\ 2i & -3+4i & 4-3i \\ -2 & 5 & 3 \end{pmatrix}$$

The Conjugate Transpose T^*

Definition

If $T = (a_{ij})$ is a square complex matrix, then we say that $T^* = (\overline{a_{ji}})$ is its conjugate transpose.

$$T = \begin{pmatrix} 1 & 6 & i \\ 2i & -3+4i & 4-3i \\ -2 & 5 & 3 \end{pmatrix} \quad T^* = \begin{pmatrix} 1 & -2i & -2 \\ 6 & -3-4i & 5 \\ -i & 4+3i & 3 \end{pmatrix}$$

Conjugations

Conjugations

Definition

A conjugation C is a isometric antilinear involution.

Conjugations

Definition

A conjugation C is a isometric antilinear involution.

- Isometric: leaves sizes and angles unchanged.

Conjugations

Definition

A conjugation C is a isometric antilinear involution.

- Isometric: leaves sizes and angles unchanged.
- Antilinear: $C(\lambda x) = \overline{\lambda} Cx$.

Conjugations

Definition

A conjugation C is a isometric antilinear involution.

- Isometric: leaves sizes and angles unchanged.
- Antilinear: $C(\lambda x) = \overline{\lambda} Cx$.
- Involution: $C \circ C = I$.

Conjugations

Definition

A conjugation C is a isometric antilinear involution.

- Isometric: leaves sizes and angles unchanged.
- Antilinear: $C(\lambda x) = \overline{\lambda} Cx$.
- Involution: $C \circ C = I$.

Definition

The standard conjugation J takes a vector to its conjugate:

$$J(x_1, x_2, \ldots, x_n) = (\overline{x_1}, \overline{x_2}, \ldots, \overline{x_n}).$$

C-symmetry

C-symmetry

Definition

We say a matrix T is C-symmetric if there exists a conjugation C such that $T = CT^*C$.

C-symmetry

Definition

We say a matrix T is C-symmetric if there exists a conjugation C such that $T = CT^*C$.

Theorem

A matrix is UECSM if and only if it is C-symmetric for some conjugation C.

Derivative Results

Derivative Results

- Every 2×2 matrix is UECSM.

Derivative Results

- Every 2×2 matrix is UECSM.
- Rank 1 matrices are UECSM.

Derivative Results

- Every 2×2 matrix is UECSM.
- Rank 1 matrices are UECSM.
- Direct sum of UECSM is UECSM.

A Brief Review of Eigenvectors

A Brief Review of Eigenvectors

Definition

Let T be a matrix. Then if there exists a vector v and a scalar λ such that $Tv - \lambda v = 0$, then we say that λ is an eigenvalue of T and v is an eigenvector with eigenvalue λ.

A Brief Review of Eigenvectors

Definition

Let T be a matrix. Then if there exists a vector v and a scalar λ such that $Tv - \lambda v = 0$, then we say that λ is an eigenvalue of T and v is an eigenvector with eigenvalue λ.

Definition

Let T, v, λ be as above. If there exists a natural number n such that $(T - \lambda I)^n v = 0$ then v is a generalized eigenvector of T with eigenvalue λ.

Conjugating Eigenvectors

Conjugating Eigenvectors

Fact

T and T^* have conjugate eigenvalues.

Conjugating Eigenvectors

Fact

T and T^* have conjugate eigenvalues.

$$Tv = \lambda v \Rightarrow T^* u = \overline{\lambda} u.$$

Conjugating Eigenvectors

Fact

T and T^* have conjugate eigenvalues.

$$Tv = \lambda v \Rightarrow T^* u = \overline{\lambda} u.$$

$$\lambda v = Tv = CT^* Cv$$

$$\overline{\lambda}(Cv) = C(\lambda v) = CTv = T^*(Cv)$$

Thus Cv is an eigenvector of T^* with eigenvalue $\overline{\lambda}$.

Conjugating Eigenvectors

Fact

T and T^* have conjugate eigenvalues.

$$Tv = \lambda v \Rightarrow T^* u = \overline{\lambda} u.$$

$$\lambda v = Tv = CT^* Cv$$

$$\overline{\lambda}(Cv) = C(\lambda v) = CTv = T^*(Cv)$$

Thus Cv is an eigenvector of T^* with eigenvalue $\overline{\lambda}$.

Thus C must take eigenvectors of T to corresponding eigenvectors of T^*.

How to Classify?

How to Classify?

- Unitary Equivalence is an equivalence relation.

How to Classify?

- Unitary Equivalence is an equivalence relation.
- Which representative should we use?

How to Classify?

- Unitary Equivalence is an equivalence relation.
- Which representative should we use?

Schur's Theorem

Every square matrix is unitarily equivalent to an upper triangular matrix.

How to Classify?

- Unitary Equivalence is an equivalence relation.
- Which representative should we use?

Schur's Theorem

Every square matrix is unitarily equivalent to an upper triangular matrix.

$$\begin{pmatrix} \lambda_1 & a & b \\ 0 & \lambda_2 & c \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

A (Pseudo)-General Algorithm

A (Pseudo)-General Algorithm

$$T = \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A (Pseudo)-General Algorithm

$$T = \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$u_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, u_1 = \begin{pmatrix} b \\ 0 \\ 1 \end{pmatrix}, v_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

A (Pseudo)-General Algorithm

$$T = \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$u_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, u_1 = \begin{pmatrix} b \\ 0 \\ 1 \end{pmatrix}, v_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$|b + 0 + 0| = |0 + 0 + 0|$$

A (Pseudo)-General Algorithm

$$T = \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$u_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, u_1 = \begin{pmatrix} b \\ 0 \\ 1 \end{pmatrix}, v_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$|b + 0 + 0| = |0 + 0 + 0|$

$u_0 \longrightarrow v_0, \; u_1 \longrightarrow v_1, \; u_\lambda \longrightarrow v_\lambda.$

Our Cases

Our Cases

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$$

Our Cases

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & a & b \\ 0 & 1 & c \\ 0 & 0 & \lambda \end{pmatrix}$$

One Eigenvalue

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$$

One Eigenvalue

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$$

Rank 1

One Eigenvalue

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$$

Rank 1

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Rank 1

One Eigenvalue

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$$
Rank 1

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
Rank 1

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$$
$|a| = |c|$

Two Eigenvalues

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{pmatrix}$$

Two Eigenvalues

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underset{\text{Rank } 1}{\begin{pmatrix} 0 & 0 & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{pmatrix}} \qquad \underset{2 \times 2 \oplus 1 \times 1}{\left(\begin{array}{cc|c} 0 & a & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)}$$

Two Eigenvalues

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{pmatrix}$$
Rank 1

$$\left(\begin{array}{cc|c} 0 & a & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$
$2 \times 2 \oplus 1 \times 1$

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
Angle Test

Two Eigenvalues

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{pmatrix} \quad \left(\begin{array}{cc|c} 0 & a & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 1 \end{array}\right) \quad \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rank 1 $2 \times 2 \oplus 1 \times 1$ Angle Test

$$\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & c \\ 0 & 0 & 1 \end{pmatrix}$$

Three Eigenvalues

$$\begin{pmatrix} 0 & a & b \\ 0 & 1 & c \\ 0 & 0 & \lambda \end{pmatrix}$$

Three Eigenvalues

$$\begin{pmatrix} 0 & a & b \\ 0 & 1 & c \\ 0 & 0 & \lambda \end{pmatrix}$$

$$\left(\begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & 1 & c \\ 0 & 0 & \lambda \end{array}\right) \qquad \left(\begin{array}{cc|c} 0 & a & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & \lambda \end{array}\right) \qquad \begin{pmatrix} 0 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

$$2 \times 2 \qquad 2 \times 2 \oplus 1 \times 1 \qquad 2 \times 2 \oplus 1 \times 1$$

Three Eigenvalues

$$\begin{pmatrix} 0 & a & b \\ 0 & 1 & c \\ 0 & 0 & \lambda \end{pmatrix}$$

$$\left(\begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & 1 & c \\ 0 & 0 & \lambda \end{array}\right) \qquad \left(\begin{array}{cc|c} 0 & a & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & \lambda \end{array}\right) \qquad \begin{pmatrix} 0 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

2×2 $\quad$ $2 \times 2 \oplus 1 \times 1$ $\quad$ $2 \times 2 \oplus 1 \times 1$

$$\begin{pmatrix} 0 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & b \\ 0 & 1 & c \\ 0 & 0 & \lambda \end{pmatrix}$$

Angle Test $\quad$ Angle Test

Three Eigenvalues

$$\begin{pmatrix} 0 & a & b \\ 0 & 1 & c \\ 0 & 0 & \lambda \end{pmatrix}$$

$$\left(\begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & 1 & c \\ 0 & 0 & \lambda \end{array}\right) \qquad \left(\begin{array}{cc|c} 0 & a & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & \lambda \end{array}\right) \qquad \begin{pmatrix} 0 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

2×2 $\qquad$ $2 \times 2 \oplus 1 \times 1$ $\qquad$ $2 \times 2 \oplus 1 \times 1$

$$\begin{pmatrix} 0 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & b \\ 0 & 1 & c \\ 0 & 0 & \lambda \end{pmatrix}$$

Angle Test $\qquad$ Angle Test

Partial Isometries

Definition

A matrix T is a partial isometry if there exists a unitary matrix U and an orthogonal projection P such that $T = UP$.

Partial Isometries

Definition

A matrix T is a partial isometry if there exists a unitary matrix U and an orthogonal projection P such that $T = UP$.

Proposition

Every 3×3 partial isometry is UECSM.

Partial Isometries

Definition

A matrix T is a partial isometry if there exists a unitary matrix U and an orthogonal projection P such that $T = UP$.

Proposition

Every 3×3 partial isometry is UECSM.

Conjecture

Every 3×3 UECSM is a rank 1 matrix, a $2 \times 2 \oplus 1 \times 1$, or some multiple of a partial isometry, plus some multiple of the identity matrix.


damn anti-gravity cat
always disproving ma theorem
ICANHASCHEEZBURGER.COM

$$
\langle 2*u111*u211+2*u112*u212+2*u121*u221+2*u122*u222+2*u131*u231+2*u132*u232,\ 2*u111*u311+2*u112*u312+2*u121*u321+2*u122*u322+2*u131*u331+2*u132*u332,\ 2*u211*u311+2*u212*u312+2*u221*u321+2*u222*u322+2*u231*u331+2*u232*u332,\ 2*u111*u212+2*u131*u232-2*u122*u221+2*u121*u222-2*u112*u211-2*u132*u231,\ 2*u121*u322-2*u112*u311-2*u132*u331+2*u111*u312+2*u131*u332-2*u122*u321,\ -2*u222*u321+2*u221*u322-2*u212*u311-2*u232*u331+2*u211*u312+2*u231*u332,\ 1-u111^2-u112^2-u121^2-u122^2-u131^2-u132^2,\ 1-u211^2-u212^2-u221^2-u222^2-u231^2-u232^2,\ 1-u311^2-u312^2-u321^2-u322^2-u331^2-u332^2,\ -2*s111*u111+2*s112*u112-2*s121*u211+2*s122*u212-2*s131*u311+2*s132*u312,\ 2*u111*a1-2*s111*u121-2*u112*a2-2*s121*u221+2*s112*u122+2*s122*u222-2*s131*u321+2*s132*u322+2*u121,\ 2*u111*b1+2*u121*c1-2*u112*b2-2*u122*c2+2*u131*q1-2*u132*q2-2*s111*u131-2*s121*u231+2*s112*u132+2*s122*u232-2*s131*u331+2*s132*u332,\ -2*s121*u111+2*s122*u112-2*s221*u211+2*s222*u212-2*s231*u311+2*s232*u312,\ -2*u212*a2+2*u211*a1+2*s222*u222-2*s221*u221+2*s122*u122-2*s121*u121+2*s232*u322-2*s231*u321+2*u221,\ 2*u211*b1-2*u212*b2-2*s121*u131-2*u232*q2+2*u231*q1-2*u222*c2+2*u221*c1-2*s231*u331+2*s222*u232-2*s221*u231+2*s122*u132+2*s232*u332,\ -2*s131*u111+2*s132*u112-2*s231*u211+2*s232*u212-2*s331*u311+2*s332*u312,\ 2*s132*u122-2*s131*u121-2*s231*u221+2*s232*u222-2*s331*u321+2*s332*u322+2*u321+2*u311*a1-2*u312*a2,\ -2*u322*c2-2*u332*q2+2*u331*q1-2*s131*u131+2*s232*u232-2*s231*u231+2*s132*u132-2*s331*u331+2*s332*u332+2*u321*c1+2*u311*b1-2*u312*b2,\ 2*s112*u111+2*s122*u211+2*s131*u312+2*s121*u212+2*s111*u112+2*s132*u311,\ -2*u122-2*u111*a2+2*s111*u122-2*u112*a1+2*s131*u322+2*s132*u321+2*s112*u121+2*s122*u221+2*s121*u222,\ -2*u132*q1-2*u131*q2-2*u111*b2-2*u112*b1-2*u121*c2-2*u122*c1+2*s111*u132+2*s112*u131+2*s121*u232+2*s122*u231+2*s131*u332+2*s132*u331,\ 2*s222*u211+2*s221*u212+2*s122*u111+2*s121*u112+2*s231*u312+2*s232*u311,\ 2*s231*u322-2*u222+2*s222*u221-2*u211*a2-2*u212*a1+2*s121*u122+2*s122*u121+2*s221*u222+2*s232*u321,\ -2*u231*q2-2*u211*b2-2*u221*c2+2*s221*u232-2*u232*q1-2*u222*c1+2*s231*u332+2*s222*u231-2*u212*b1+2*s121*u132+2*s122*u131+2*s232*u331,\ 2*s131*u112+2*s331*u312+2*s232*u211+2*s132*u111+2*s231*u212+2*s332*u311,\ -2*u322-2*u311*a2-2*u312*a1+2*s131*u122+2*s132*u121+2*s231*u222+2*s232*u221+2*s331*u322+2*s332*u321,\ -2*u322*c1-2*u321*c2-2*u331*q2+2*s231*u232+2*s232*u231+2*s331*u332+2*s332*u331-2*u312*b1-2*u332*q1+2*s131*u132+2*s132*u131-2*u311*b2\rangle
$$


goals
i haz dem
ICANHASCHEEZBURGER.COM

To Paraphrase Richard Feynman:

Math is like sex. Sure, it may give some practical results, but that's not why we do it.