Lab 2 Thursday September 8

Visualizing limits

Recall from last week that we can plot a function `f[x]`, on the domain $[a, b]$, with the command `Plot[f[x],{x,a,b}]` If we want to show only the range from s to t we can use the command `Plot[f[x],{x,a,b},PlotRange->{s,t}]`

We will begin by studying limits of the function x^2.

1. Plot the function x^2 around the point $a = 0$ with the command `Plot[x^2, {x,-2,2}]`

2. Based on this picture, estimate $\lim_{x\to 0} x^2$.

3. Recall the definition of a limit: for every error ϵ we should be able to find a distance δ so that if $|x - a| < \delta$ then $|f(x) - L| < \epsilon$.

   For now, let's set the error margin to $\epsilon = 1$. If we plot `Plot[x^2, {x,-2,2}, PlotRange->{-1,1}]` we can only see outputs that are within our error margin–our range is set to within 1 unit of 0.

   Based on this picture, if our input is between -2 and 2, will our output be within our error margin? What is our δ here?

4. What does δ need to be to make our output land in our error margin? Plot another graph with the same `PlotRange` but a smaller domain so that all your outputs are within the error margin.

   If you're having trouble telling whether all your outputs are within the error margin, add the option `Filling->Automatic`.

5. If we use an error margin of $\epsilon = 1/4$, what δ do we need? Plot the corresponding graph.

6. Plot another graph for $\epsilon = 1/10$.

7. Come up with a formula for what δ needs to be in terms of ϵ. (hint: check your notes). Then use the following code:

   ```
   epsilon = 1

   delta = Sqrt[epsilon]

   Plot[x^2,{x,0-delta,0+delta},PlotRange->{0-epsilon,0+epsilon}
   ```

   Run this code with several different values of ϵ. Does it work every time?

8. (Optional) Run the code

   ```
   Table[Plot[x^2,{x,0-Sqrt[epsilon],0+Sqrt[epsilon]},Filling->Automatic,

   PlotRange->{0-epsilon,0+epsilon}],{epsilon,{1,1/2,1/4,1/10,1/100}}]
   ```

   to see graphs for a variety of different ϵ. (You can grab a copy of this worksheet from the course web page and copy and paste the code).

Exercises

Below there is a list of functions f paired with numbers a. For each item of the list:

1. Plot a graph of f centered at the point a.
2. Use this graph to estimate $L = \lim_{x \to a} f(x)$.
3. Plot a graph with `PlotRange` given by $\epsilon = 1$–that is, `PlotRange->{L-epsilon,L+epsilon}`. What δ do we need to make all outputs fall within ϵ of L?
4. Do the same with $\epsilon = 1/10$.
5. Find a formula for δ in terms of ϵ and test it for a few different ϵ.
6. (Optional) Make a table for graphs at different values of ϵ.

(a) $f(x) = x^2, a = 1$

(b) $f(x) = 2x, a = -2$

(c) $f(x) = 1/x, a = 1$

(d) $f(x) = 1/x, a = 10$

(e) $f(x) = x^2 + 3, a = 0$

(f) $f(x) = \frac{x^2-4}{x-2}, a = 2$

(g) $f(x) = x^3 + x, a = 1$

Bonus: $f(x) = \sin(x), a = 0$