

## Lab 4

Thursday September 22

## The Squeeze Theorem

1. As a refresher, plot  $x * \text{Sin}[1/x]$  on the same graph as  $\text{Abs}[x]$  and  $-\text{Abs}[x]$ , with domain  $[-1, 1]$  with the command

```
Plot[{x * Sin[1/x], Abs[x], -Abs[x]}, {x, -1, 1}]
```

(Bonus: color-code the graph so  $x*\text{Sin}[1/x]$  is one color and the other two graphs are a different color, using the `PlotStyle` command). Shrink the domain around the point  $x = 0$  to see what happens. What does this tell us about the limit?

2. Tomorrow we will discuss the important limit  $\text{Sin}[x]/x$ , again using the squeeze theorem. As in (1), plot a graph of  $\text{Sin}[x]/x$ , along with 1 and  $\text{Cos}[x]$ . (Again, for a bonus, color-code your graph). Shrink the domain around  $x = 0$  to see what happens.

3. In class we discussed the function  $s[x_] := (x-1)/(2+\text{Sin}[1/(x - 1)])$

- Plot the function  $s$  on the domain  $[0, 2]$ . Now plot it on the domain  $].9, 1.9]$ .
- Plot  $s$  together with the functions  $x - 1$  and  $(x - 1)/3$ . Is  $s$  squeezed between the two functions?
- Now plot  $s$  together with  $\text{Abs}[x-1]$  and  $\text{Abs}[x-1]/3$ . Is the function trapped here?
- Try the pairs  $\text{Abs}[x-1]$ ,  $-\text{Abs}[x-1]/3$  and  $\text{Abs}[x-1]$ ,  $-\text{Abs}[x-1]$ . What happens?

4. We can evaluate limits of functions like  $f[x_] := x/(x^2 + 1)$  straightforwardly, but we can also use these techniques of trapping it between two other curves.

Our goal is to find two functions, one of which is *always* bigger than  $f$  and the other of which is *always* smaller, that have the same output at 0.

- What is the *smallest* the denominator can get?
- What does this imply about the *biggest* the function  $f$  can get? Can we write a function that's always bigger? Graph  $f$  and your guess together and check that your guess is always bigger.
- Come up with upper and lower bounds on the function  $f$  that it is squeezed between.

Then plot a graph showing  $f$  squeezed in between your two boundary functions at  $x = 0$ .

5. Do the same thing with  $g[x_] := x^2/(1 + \text{Sin}[(x^2+12)/(x^3)]^2)$

(Hint: this is  $\frac{x^2}{1 + \sin^2\left(\frac{x^2+12}{x^3}\right)}$  )

## Small Angle Approximation

1. `Plot[{Sin[x],x},{x,-1,1}]`
2. `Plot[{Sin[5x],5x},{x,-1,1}]`
3. `Plot[{Sin[x^2-4],x^2-4},{x,1,3}]`
4. `Plot[{Sin[Tan[x]],Tan[x]},{x,-1,1}]`

## Filling in functions

We can always take a function whose domain is not all reals and extend it to a function whose domain is all real numbers by simply assigning values to all the points that aren't already covered. You can think of this as defining a piecewise function where one of the pieces is just a single point. We often want to extend a given function to be a *continuous* function on all reals; however, this is not always possible.

Go to the course webpage and download the file called “plot piecewise file”. If the computer asks to open it with Mathematica, do that; if not, copy all the text (use ctrl+a) and then paste it into an empty Mathematica notebook, and when asked if you want to interpret the text, click “yes”. Click anywhere in the giant block of code and hit shift+enter; this gives you the new function `PlotPiecewise` which we will be using for the rest of this lab.

For each of the following functions, do the following steps:

- (a) Figure out the domain, and define the function `f[x]` in mathematica.
- (b) Plot the function using the `PlotPiecewise` command from the notebook I posted on Moodle, and the command `PlotPiecewise[f[x],{x,-3,3}]`  
Did you get the domain right? (Notice that for sufficiently strange functions this algorithm will not work properly).
- (c) Can you assign values to the function at the holes so that the function is “nice” or “connected” (continuous)? (“No” is a possible answer here, but think about why).
- (d) If you can, define a new function `fFixed[x]` that has the holes filled in, using the `Piecewise` command. E.g. if there is a hole at  $x = 2$  and you think the value “should” be 5, enter `fFixed[x_] := Piecewise[{{f[x],x!= 2},{5,x==2}}`  
(note we use `!=` for “not equals” and `==` for “equals” )
- (e) Plot your new function in `PlotPiecewise` to see if you have successfully filled the holes.  
`PlotPiecewise[fFixed[x],{x,-3,3}]`

1. `f[x_] := (x^2-1)/(x-1)`
2. `g[x_] := 1/(x+1)`
3. `h[x_] := (x+1)^2/(x^2-1)`
4. `i[x_] := (x^2-2x-1)/(x-1)`
5. `j[x_] := 2 Abs[x+1]/(2x+2)`
6. `k[x_] := Sin[1/x]`
7. `l[x_] := x * Sin[1/x]`
8. `m[x_] := Sin[x-1]/(x-1)`
9. `n[x_] := (x^3-8)/(x^2-4)`
10. `o[x_] := (x^2-x)/(x^3-3x^2+2x)`