Lab 8 Thursday October 27

Implict Functions and their Tangents

When using the `ContourPlot` command, note the double `==` signs.

1. We showed that the tangent line to $x^3 + y^3 = 6xy$ at the point $(3, 3)$ is $y - 3 = 3 - x$. Verify this with the command
   `ContourPlot[{x^3 + y ^3 == 6 x*y, y-3 == 3 -x}, {x,-5,5},{y,-5,5}]`

2. (a) Use ContourPlot to plot the "cardioid" with equation:
   `x^2 + y^2 == ( 2x^2 + 2y^2 -x)^2`. (x and y domains from -1 to 1).
   (b) Compute the derivative at the point $(0, 1/2)$ by hand.
   (c) Check your computation by running the commands
   `D[x^2 + y[x]^2 == (2x^2 + 2y[x]^2 - x)^2,x]` and
   `D[x^2 + y[x]^2 == (2x^2 + 2y[x]^2 - x)^2,x] /. y[x] -> 1/2 /.x -> 0`
   Note some important details here. Mathematica can't figure out that `y` is a function of `x` instead of a constant unless we tell it, so we write `y[x]` instead of `y`. We can have mathematica automatically substitute for us, but it matters that we do `y[x]` before `x`. Why? Try it the other way and see what happens.
   (d) Plot the tangent line to the cardioid at that point in Mathematica.
   (e) What do you expect to happen if you try to find the tangent line at $(0, 0)$? Are you right? What does Mathematica say?

3. (a) Plot the "devil's curve" `y^2(y^2 -4) == x^2 (x^2 - 5)`
   (b) Compute the derivative at $(0, -2)$ by hand.
   (c) Use Mathematica to check your answer.
   (d) Plot the devil's curve and its tangent line simultaneously.
   (e) (Just for fun) Run the command
   `ContourPlot[y^2(y^2-4) - x^2(x^2 -5),{x,-5,5},{y,-5,5}]` What happens? Why?

4. (a) Plot `(x^2 + y^2 -1)^3 - x^2 * y^3 == 0`
   (b) Check that $(1, 1)$ is a solution to this equation, and compute the derivative at $(1, 1)$. Use Mathematica to check your answer.
   (c) Plot the tangent line.
   (d) (Just for fun) Now try plotting without the equals sign, as in (3).

5. (a) Plot `Sin[x^2 + y^2] == Cos[x * y]` from -5 to 5.
   (b) Find the derivative at $(\sqrt{\pi/6}, \sqrt{\pi/6})$. (I know that looks terrible, but in context it's actually really easy to compute). Plot the tangent line.
   (c) (Just for fun) As before, replace the `==` with a `-` sign.

Bonus: Just for fun and pretty

1. Some other functions to try:

   - `Sin[Sin[x] + Cos[y]] == Cos[Sin[x * y] + Cos[y]]`
   - `Abs[Sin[x^2 - y^2]] == Sin[x + y] + Cos[x * y]`
   - `Csc[1-x^2] * Cot[2-y^2] == x * y`
   - `Abs[Sin[x^2 + 2 * x * y]] == Sin[x - 2 y]`
   - `(x^2 + y^2 - 3) Sqrt[x^2 + y^2] + .75 + Sin[4 Sqrt[x^2  + y^2]] Cos[84 ArcTan[y/x]] - Cos[6 ArcTan[y/x]] ==  0`

2. Try replacing the == signs with - signs.

3. Look at the examples on the Wolfram Alpha page `https://www.wolframalpha.com/examples/PopularCurves.html`

4. Search google for interesting pictures from implicit curves.