Math 322 Fall 2016
Number Theory HW 11
Due Wednesday, November 30

There is no starred problem this week!

For the remainder of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. Let $k \geq 3$ be an integer. Then prove that

$$\operatorname{ord}_{2^k} 5 = \phi(2^k)/2 = 2^{k-2}.$$

(Hint: we know that the order divides 2^{k-1} and is not equal to 2^{k-1}; the largest it can possibly be is 2^{k-2}. So we just have to prove it's no smaller).

2. (6 points total)

Let m be a natural number with primitive root r, and let a, b be relatively prime to m. Then prove that:

   (a) $\operatorname{ind}_r 1 \equiv 0 \mod \phi(m)$
   (b) $\operatorname{ind}_r(ab) \equiv \operatorname{ind}_r a + \operatorname{ind}_r b \mod \phi(m)$
   (c) $\operatorname{ind}_r a^k \equiv k \operatorname{ind}_r a \mod \phi(m)$.

3. Find all solutions to $7x^9 \equiv 4 \mod 17$.

4. Find all solutions to $5^x \equiv 4 \mod 17$.

5. (a) Compute the index base 2 of 15 modulo 19.
   (b) Compute the index base 3 of 15 modulo 19.
   (c) Find all natural numbers less than 13 which are squares modulo 13.
   (d) Find all natural numbers less than 13 which are cubes modulo 13.