Problem 1. (a) Use the definition of limit to prove that $\lim_{x\to 2} \frac{1}{x+3} = \frac{1}{5}$.

(b) Use the definition of limit to prove that $\lim_{x\to 1} \frac{1}{(x-1)^2} = +\infty$.

Problem 2. (a) Use the Squeeze Theorem to show that $\lim_{x\to 5}(x-5)\sin\left(\frac{x^2+1}{x-5}\right) = 0$.

(b) Compute $\displaystyle\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25}$

(c) Compute $\displaystyle\lim_{x\to 0} \frac{\sin(x^2)}{\sin^2(x)}$

Problem 3. (a) **Directly from the definition**, compute $f'(1)$ where $f(x) = \sqrt{x+3}$.

(b) Compute $g'(x)$ where $g(x) = \ln \left| \dfrac{e^{\arctan(x^2)} - 5}{\sqrt[4]{x^2+1}} \right|$.

(c) Find a tangent line to the function $f(x) = \frac{e^x}{x}$ at the point given by $x = 2$.

Problem 4. (a) **Directly from the definition**, compute $f'(x)$ where $f(x) = \frac{1}{x-7}$.

(b) Write a tangent line to the curve $y^2 = x^{x\cos(x)}$ at the point $(\pi/2, -1)$.

(c) Find y' if $e^y + \ln(y) = x^2 + 1$.

Problem 5. (a) A cone with height h and base radius r has volume $\frac{1}{3}\pi r^2 h$. Suppose we have an inverted conical water tank, of height 4m and radius 6m. Water is leaking out of a small hole at the bottom of the tank. If the current water level is 2m and the water level is dropping at $\frac{1}{9\pi}$ meters per minute, what volume of water leaks out every minute?

(b) Use two iterations of Newton's method, starting at 0, to estimate the root of $e^x - 3x$.

(c) Let $g(x) = \sqrt[5]{x^9 + x^7 + x + 1}$. Find $(g^{-1})'(1)$.

Problem 6. (a) If $f(x) = \sqrt{x} + \tan(\pi x)$, use a linear approximation centered at 4 to estimate $f(4.1)$.

(b) If $g(x) = \cos(x)$, use a quadratic approximation centered at 0 to estimate $g(.1)$.

(c) Let $g'(x) = g(x) + 3x$, and $g(2) = 4$. Use two steps of Euler's method to estimate $g(4)$. Is this an overestimate or an underestimate?

Problem 7. (a) Find the absolute extrema of $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$ on $[0, 5]$.

(b) Find all the critical points of $g(x) = \ln(x^3 + 9x^2 + 27x)$.

(c) Classify the relative extrema of $h(x) = \sqrt[3]{x}(x + 4)$

Problem 8. (a) Find all the critical points of $g(x) = \dfrac{x^2 - 8}{x + 3}$

(b) If $-1 \leq f'(x) \leq 3$ and $f(0) = 0$, what can you say about $f(4)$? Assume f is continuous and differentiable.

(c) Prove that $x^2 - (e^2 + 1)\ln(x)$ has at least two real roots.

Problem 9. Let $j(x) = x^4 - 14x^2 + 24x + 6$. We can compute $j'(x) = 4(x+3)(x-1)(x-2)$ and $j''(x) = 4(3x^2 - 7)$. Sketch a graph of j.

Problem 10. Let $g(x) = \arctan(x^2 + x)$. We can compute that $g'(x) = \frac{2x+1}{1+(x^2+x)^2}$ and

$$g''(x) = \frac{-6x^4 - 12x^3 - 8x^2 - 2x + 2}{(1 + (x^2 + x)^2)^2}.$$

Sketch a graph of g.