Math 114 Practice Test 2

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Problem 1.

Compute the following limits, showing each step and naming each limit law you use.

(a)

$$\lim_{x\to 4} \sqrt{x^2 - x - 3} + \frac{2}{x}$$

(b)

$$\lim_{x\to 1} \frac{x^2 + 4x - 5}{x - 1}$$

Problem 2.

Compute the following limits if they exist. Show enough work to justify your computation, or your claim that the limit does not exist.

(a)

$$\lim_{x\to -2} \frac{x^2+6x+8}{2(x+4)(x+2)} =$$

(b)

$$\lim_{x\to 9} \frac{3-\sqrt{x}}{9-x}$$

(c)

$$\lim_{x\to -\infty} \frac{3x^3+\sqrt[3]{x}}{\sqrt{9x^6+2x^2+1}+x}$$

(d)

$$\lim_{x\to 1^+} \frac{|x-1|}{x-1} =$$

Problem 3. Compute the following limits if they exist. Show enough work to justify your computation, or your claim that the limit does not exist.

(a)

$$\lim_{x \to 1} \frac{\sin^2(x-1)}{(x-1)^2} =$$

(b)

$$\lim_{x \to -2} \frac{x^2 + 6x + 9}{2(x+4)(x+2)} =$$

(c) Using the Squeeze Theorem, show that

$$\lim_{x \to 3} \frac{x-3}{1 + \sin^2\left(\frac{2\pi + e + 7}{x-3}\right)} = 0.$$

Problem 4. (a) Show that the polynomial $x^4 - 6x - 2$ has two real roots, that is, there are two (different!) real numbers a and b such that $a^4 - 6a - 2 = b^4 - 6b - 2 = 0$.

(b) Let

$$g(x) = \begin{cases} \frac{x^2-1}{x-1} & x > 0 \\ x^2 + 1 & x < 0 \end{cases}$$

If possible, define an extension of g that is continuous at all real numbers.

Problem 5. Compute the following derivatives using only the definition of derivative.

(a) Derivative of $f(x) = x^2 + \sqrt{x}$ at $x = 2$.

(b) Derivative of $g(x) = \frac{1}{x+1}$ at $x = 1$.