Problem 1. (a) Use the definition of limit to prove that $\lim_{x\to 2} \frac{1}{x+3} = \frac{1}{5}$.

(b) Use the definition of limit to prove that $\lim_{x\to 1} \frac{1}{(x-1)^2} = +\infty$.

Problem 2. (a) Use the Squeeze Theorem to show that $\lim_{x\to 5} (x-5) \sin\left(\frac{x^2+1}{x-5}\right) = 0$.

(b) Compute
$$\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25}$$

(c) Compute $\lim_{x\to 5} \frac{e^{x-5}-1}{x-5}$

Problem 3. (a) Directly from the definition, compute f'(1) where $f(x) = \sqrt{x+3}$.

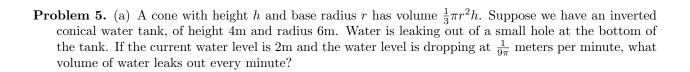
(b) Compute
$$g'(x)$$
 where $g(x) = \ln \left| \frac{e^{\arctan(x^2)} - 5}{\sqrt[4]{x^2 + 1}} \right|$.

(c) Find a tangent line to the function $f(x) = \frac{e^x}{x}$ at the point given by x = 2.

Problem 4. (a) Directly from the definition, compute f'(x) where $f(x) = \frac{1}{x-7}$.

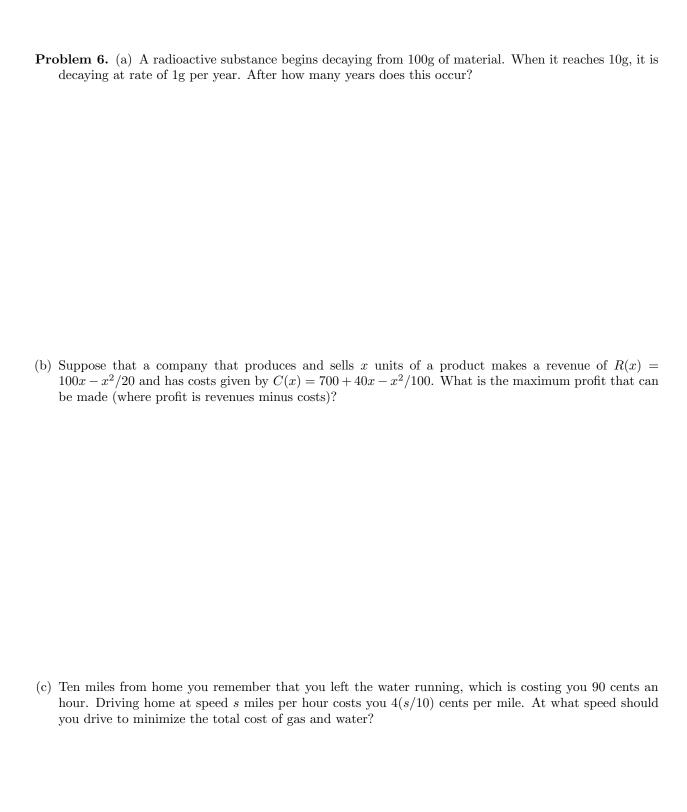
(b) Write a tangent line to the curve $y^2 = x^{x\cos(x)}$ at the point $(\pi/2, -1)$.

(c) Find y' if $e^y + \ln(y) = x^2 + 1$.



(b) Use two iterations of Newton's method, starting at 0, to estimate the root of $e^x - 3x$.

(c) Let g'(x) = g(x) + 3x, and g(2) = 4. Use two steps of Euler's method to estimate g(4). Is this an overestimate or an underestimate?



Problem 7. (a) Find the absolute extrema of $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$ on [0, 5].

(b) Find all the critical points of $f(x) = \frac{\ln(x)}{x^2 - 3x - 2}$

(c) Classify the relative extrema of $h(x) = \sqrt[3]{x}(x+4)$

Problem 8. (a) Find all the critical points of $g(x) = \frac{x^2 - 8}{x + 3}$

(b) If $-1 \le f'(x) \le 3$ and f(0) = 0, what can you say about f(4)? Assume f is continuous and differentiable.

(c) Prove that $x^2 - (e^2 + 1) \ln(x)$ has exactly two real roots.

Problem 9. Sketch the graph of $j(x) = x^4 - 14x^2 + 24x + 6$. (Don't worry about finding roots).

Problem 10. Sketch a graph of the function $h(x) = e^x - \ln(|x|)$.