Math 114 Practice Test 2

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Problem 1. Compute the following limits if they exist. Show enough work to justify your computation, or your claim that the limit does not exist.

(a)

$$\lim_{x \to 1} \frac{\sin^2(x-1)}{(x-1)^2} =$$

(b)

$$\lim_{x \to -2} \frac{x^2+6x+9}{2(x+4)(x+2)} =$$

(c) Using the Squeeze Theorem, show that

$$\lim_{x \to 3} \frac{x-3}{1+\sin^2\left(\frac{2\pi+e+7}{x-3}\right)} = 0.$$

Problem 2. (a) Show that the polynomial $x^4 - 6x - 2$ has two real roots, that is, there are two (different!) real numbers a and b such that $a^4 - 6a - 2 = b^4 - 6b - 2 = 0$.

(b) Let

$$g(x) = \begin{cases} \frac{x^2-1}{x-1} & x > 0 \\ x^2 + 1 & x < 0 \end{cases}$$

If possible, define an extension of g that is continuous at all real numbers.

Problem 3. Compute the following derivatives using only the definition of derivative.

(a) Derivative of $f(x) = x^2 + \sqrt{x}$ at $x = 2$.

(b) Derivative of $g(x) = \frac{1}{x+1}$ at $x = 1$.

Problem 4. You may use any methods we have learned in class to solve these problems, but show enough work to justify your answers.

(a) Find $\frac{d^2f}{dx^2}$ if $f(x) = x \cos x$.

(b) If $g(x) = \sin(3x)$ compute $g'(\pi/12)$

(c) Find an equation of the line tangent to $y = \frac{x^2-1}{x^2+1}$ at the point $(0, -1)$.