Math 114 Practice Exam 3

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Problem 1. Compute the derivatives of the following functions using methods we have learned in class. Show enough work to justify your answers.

(a) $f(x) = \sec\left(\dfrac{\sqrt{x^2+1}}{e^x+2}\right)$

(b) $g(x) = \sqrt[4]{\dfrac{x^3+\cos(x^2)}{\log_3(x+1)+1}}$

Problem 2. (a) Find a formula for y' in terms of x and y if $x^8 + x^4 + y^4 + y^6 = 1$.

(b) Compute $f'(\pi)$ where $f(x) = 3^{\sin(x)}$.

(c) Compute $g'(4)$ where $g(x) = \ln(x^3 + 3x + \sqrt{x})$.

Problem 3. (a) Let $j(x) = \sqrt[3]{x^5 + x^4 + x^3 + x^2 + 2x}$. Find $(j^{-1})'(4)$.

(b) Find a tangent line to the curve given by $x^4 - 2x^2y^2 + y^4 = 16$ at the point $(\sqrt{5}, 1)$.

Problem 4. (a) It is a fact that $2^{10} = 1024$. Estimate 2.01^{10} using the derivative of x^{10} at the point 2.

(b) Suppose we have the differential equation $f'(t) = f(t) - t$, with $f(1) = 2$. Use Euler's method with three steps to approximate $f(4)$.

(c) Use two iterations of Newton's method, starting at 2, to estimate the cube root of 9.

Problem 5. (a) Solve a stereotypical math problem: Two cars start moving from the same point. One travels south at 55 miles per hour and the other travels west at 30 miles per hour. Two hours later, how quickly is the distance between the two cars increasing?

(b) A car is driving down a road at 150 feet per second (this is about a hundred miles an hour). A camera is placed 200 feet from the road, which will rotate to follow and record the progress of the car. How quickly must the camera rotate when the car is fifty feet away from directly in front of the camera?