Math 214 Spring 2017
Linear Algebra HW 3
Due Friday, February 10

For all these problems, justify your answers; do not just write "yes" or "no".

1. (a) Can you write $(1,3,2,4)$ as a linear combination of $(1,0,1,0),(0,1,0,2)$, and $(0,1,1,1)$?
   (b) Can you write $(1,3,2,4)$ as a linear combination of $(1,0,1,0),(0,1,0,2)$, and $(1,1,1,1)$?
   (c) Write $4x+6x^3-x^5$ as a linear combination of $x+x^3, x^3+x^5$, and $x+x^5$.
2. Let $V=\mathbb{R}^3$.
   (a) Is $S=\{(1,2,3),(2,3,4),(3,4,5)\}$ a spanning set for $\mathbb{R}^3$?
   (b) Is $T=\{(1,2,3),(2,3,4),(0,1,1)\}$ a spanning set for $\mathbb{R}^3$?
3. (a) Is $S=\{(1,1,0,0),(1,-1,0,0),(0,0,1,-1),(0,0,-1,1)\}$ a spanning set for $\mathbb{R}^4$?
   (b) Is $T=\{1,1+x,1+x^2\}$ a spanning set for $\mathcal{P}_2(x)$?
4. Suppose $S=\{\mathbf{v}_1,\mathbf{v}_2,\dots,\mathbf{v}_n\}\subset V$ is a spanning set for V. Show that $T=\{\mathbf{v}_1,\mathbf{v}_2-\mathbf{v}_1,\mathbf{v}_3-\mathbf{v}_2,\dots,\mathbf{v}_n-\mathbf{v}_{n-1}\}$ is a spanning set for V.
5. (a) Is $S=\{(1,1,1),(1,1,0),(1,0,0)\}$ a linearly independent set?
   (b) Is $T=\{(1,2,3),(4,5,6),(7,8,9)\}$ a linearly independent set?
   (c) Is $U=\{(3,7,5),(2,4,2),(1,3,1)\}$ a linearly independent set?
6. (a) Is $S=\{1+x,1+x^2,x+x^2\}$ a linearly independent set?
   (b) Is $T=\{1+x,1+x^2,x-x^2\}$ a linearly independent set?
   (c) Is $U=\{\sin^2,\cos^2,1\}$ a linearly independent set?
7. ($\star$) Suppose $S=\{\mathbf{v}_1,\dots,\mathbf{v}_n\}$ is linearly independent in V, and $T=\{\mathbf{v}_1+\mathbf{w},\dots,\mathbf{v}_n+\mathbf{w}\}$ is linearly dependent in V. Show that $\mathbf{w}\in\mathrm{Span}(S)$.
8. Prove that a set $S=\{\mathbf{u},\mathbf{v}\}$ of two vectors is linearly dependent if and only if one is a scalar multiple of the other.