Math 214 Spring 2017
Linear Algebra HW 6
Due Friday, March 17

For all these problems, justify your answers.

1. Let $A = \begin{bmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{bmatrix}$. Find a basis for the rowspace, columnspace, and nullspace of A.

2. Let $B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 4 & 7 & 8 \end{bmatrix}$. Find a basis for the rowspace, columnspace, and nullspace of B.

3. Use Gaussian elimination to find a basis for the span of $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \right\}$.

4. For each of the following systems of equations, is there a solution? You don't need to find the solution if it exists, but justify your answer. (Hint: think about the column space).

   (a)
   $$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 0 & 0 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}?$$

   (b)
   $$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 1 \end{bmatrix}?$$

5. ($\star$) Find the inverse of $\begin{bmatrix} 0 & -1 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 1 & -2 & 3 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ or prove it is not invertible.

6. Find the inverse of $\begin{bmatrix} 3 & 2 & 1 & 5 \\ 2 & 4 & 3 & 8 \\ -1 & 2 & 5 & 4 \\ 4 & 8 & 9 & 17 \end{bmatrix}$ or prove it is not invertible.

7. Let $L : U \to V$ be a linear transformation. Prove that $\ker(L)$ is a subspace of U. (Hint: see problem 8 from homework 5).

8. ($\star$) Let $\mathcal{C}([a, b], \mathbb{R})$ be the space of continuous functions defined on the closed interval $[a, b]$. Prove that the function from $\mathcal{C}([a, b], \mathbb{R})$ to $\mathbb{R}$ given by $f \mapsto \int_a^b f(t)\, dt$ is a linear transformation. What is the kernel of this transformation?