Math 214 Test 2
Practice Problems

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This is not a practice test, in the sense that it is not the format I expect the test to be. It is a collection of practice problems. I will update you when I finalize the test format.

I will post at least some solutions soon. I probably won't get a full solutions document written up, though we'll see what happens.

Do the following matrix multiplication computations.

1.
$$\begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ -1 & 2 \end{bmatrix} =$$

2.
$$\begin{bmatrix} 2 & 5 \\ 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & 4 \end{bmatrix} =$$

3.
$$\begin{bmatrix} 5 & 2 & -1 \\ 1 & 0 & 3 \\ 4 & 2 & 4 \end{bmatrix} \begin{bmatrix} -3 & 0 & 1 \\ 4 & -1 & 1 \\ 0 & 2 & 3 \end{bmatrix} =$$

4.
$$\begin{bmatrix} 3 & 1 & 4 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 0 & 4 & 1 \\ 2 & 3 & 8 \end{bmatrix} =$$

For each of the following matrices, find:

(a) The reduced row echelon form.

(b) A basis for the rowspace.

(c) A basis for the columnspace.

(d) The rank.

(e) A basis for the nullspace.

(f) The nullity.

1. $\begin{bmatrix} 3 & 1 & 2 \\ -1 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$

2. $\begin{bmatrix} -2 & 4 & 1 \\ -5 & 1 & 1 \\ 3 & 3 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 6 & 2 & 3 & 1 \\ 1 & 5 & 2 & -2 \\ 4 & -4 & 1 & 3 \end{bmatrix}$

4. $\begin{bmatrix} -1 & 3 & 4 \\ 2 & 5 & 2 \\ 0 & 1 & 3 \\ 4 & 1 & -2 \end{bmatrix}$

Find the rank and nullity of the following matrices

You shouldn't need to do any actual computations here. (Hint: Rank-Nullity theorem).

1. $\begin{bmatrix} 1 & 1 & 1 & 32 & 217 & 53-e & 3^3 \\ 0 & 1 & 0 & 512 & 256 & 128 & 64 \\ 0 & 1 & 1 & 12345 & 4^{4^4} & 2 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ -1 & -2 & -3 & -4 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 4 \\ 1 & 3 & 2 & 4 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Find the inverses of the following matrices, or show they are not invertible.

1. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 0 & 4 \\ 3 & 2 & -1 \\ 1 & -4 & 3 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 1 & 3 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$

For each of the following functions

(a) Identify the domain and codomain.

(b) Determine whether it is a linear transformation.

(c) Prove your answer from part (a).

(d) If it is, find a matrix with respect to the standard basis of $\mathbb{R}^n$.

(e) If it is linear, find the kernel and image.

1. $f(x, y, z) = (3x^2, x + y, 2z - y)$.
2. $f(x, y, z) = (5x + y, z - 3x, y + z)$.
3. $f(x, y, z) = (x + y, z + y)$.
4. $f(x, y) = (x + y, x - y, 1)$.

For each of the following functions

(a) Determine whether it is a linear transformation.

(b) Prove your answer from part (a).

(c) If it is, find a matrix with respect to the given bases.

(d) If it is linear, find the kernel and image.

1. $L : \mathbb{R}^2 \to \mathbb{R}^3$ given by $L(x, y) = (x, y, x + y)$, with respect to $E = \{(1, 1), (1, -1)$ and $F = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$.
2. $L : \mathbb{R}^3 \to \mathbb{R}^2$ given by $L(x, y, z) = (x + y + z, x - y)$, with respect to $E = \{(1, 1, 1), (1, -1, 0), (0, 0, 1)\}$ and $F = \{(3, 0), (0, 2)\}$.
3. $L : \mathcal{P}_3(x) \to \mathbb{R}^2$ given by $L(f(x)) = (f(1), f(2))$, with $E = \{1, x, x^2, x^3\}$ and $F = \{(1, 0), (0, 1)\}$.
4. $L : \mathcal{P}_2(x) \to \mathcal{P}_3(x)$ given by $L(f(x)) = \int_0^x f(t)\,dt$, with $E = \{1, x, x^2\}$ and $F = \{1, x, x^2, x^3\}$.
5. $L : \mathbb{R}^3 \to \mathcal{P}_3(x)$ given by $L(a, b, c) = (a + b + c) + ax + bx^2 + cx^3$, with $E = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and $F = \{1, 1 + x, 1 + x^2, 1 + x^3\}$.
6. The function $R : \mathbb{R}^3 \to \mathbb{R}^3$ given by rotating 90 degrees counterclockwise around the z axis, and then 135 degrees counterclockwize around the x axis.

Proofs

1. Let $L : U \to V$ be a linear transformation of vector spaces, with bases E and F. Let A be the matrix of L with respect to E and F. Prove that $\mathbf{v} \in \ker(L)$ if and only if $[\mathbf{v}]_E$ is in the nullspace of A.
2. Let $L : \mathbb{R}^4 \to \mathbb{R}^2$ such that $\ker(L) = \{(x, y, z, w) : x = y, z = w\}$. Prove that the image of $\mathbb{R}^4$ is $\mathbb{R}^2$.
3. Let $A \in M_{m\times n}$ and $\mathbf{b} \in \mathbb{R}^m$. Prove that if $N(A) = \{\mathbf{0}\}$ then the equation $A\mathbf{x} = \mathbf{b}$ has at most one solution.
4. Let U, V be 2-dimensional subspaces of $\mathbb{R}^3$. On your first test you showed that the set $U \cap V = \{\mathbf{u} : \mathbf{u} \in U, \mathbf{u} \in V\}$ of vectors in both U and V is a subspace of $\mathbb{R}^3$. Prove that $\dim(U \cap V) \neq 0$. (Hint: let $\{\mathbf{u}_1, \mathbf{u}_2\}$ be a basis for U and $\{\mathbf{v}_1, \mathbf{v}_2\}$ be a basis for V. What can you say about $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{v}_1, \mathbf{v}_2\}$?