Math 114 Fall 2018
Calculus I HW 1 Solutions
Due Wednesday, September 5

1. (a) Find two real numbers that solve $x^2 + 7x + 5 = 0$. **Solution:** By the quadratic formula, we have

$$x = \frac{-7 \pm \sqrt{49 - 20}}{2} = \frac{-7 \pm \sqrt{29}}{2}.$$

   (b) Factor $x^3 - 27$.

   Solution: By the difference of cubes formula, we have $x^3 - 27 = (x^2 + 3x + 9)(x - 3)$. We can check our work by multiplying this out:

$$(x^2 + 3x + 9)(x - 3) = x^3 + 3x^3 + 9x - 3x^3 - 9x - 27 = x^3 - 27.$$

2. Based on the graphs below, estimate the following limits:

   (a) $\lim_{x \to 1} f(x)$

   (b) $\lim_{x \to -2} g(x)$

   (c) $\lim_{x \to 1} h(x)$

   (d) $\lim_{x \to 1} j(x)$

   Solution:

   (a) 0

   (b) -6

   (c) 2

   (d) 3

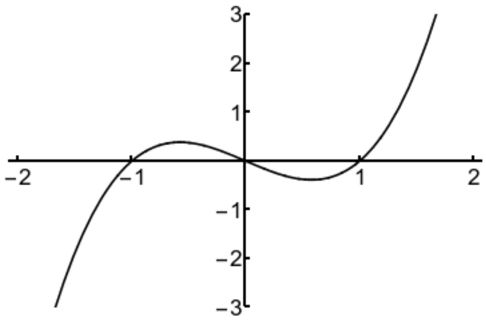

Figure 1: $f(x)$

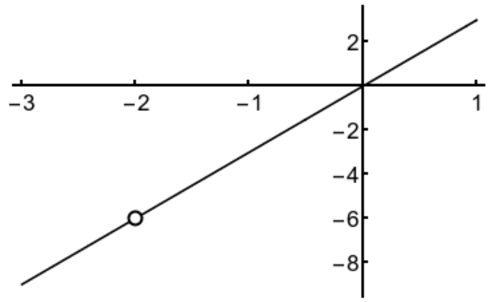

Figure 2: $g(x)$

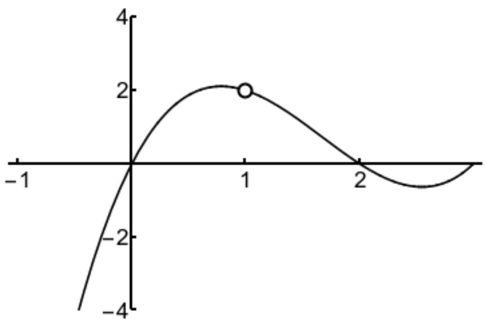

Figure 3: $h(x)$

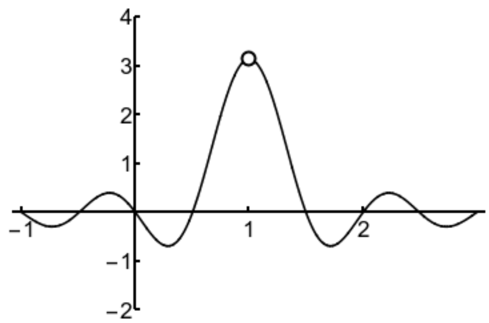

Figure 4: $j(x)$

3. If $|f(x)| \le |x|$ and $|g(x)| \le 7 + x^2$, what can we say about $|f(x) + g(x)|$ using the triangle inequalities? You should give three separate answers.

   Solution: By the triangle inequality we know that

$$|f(x) + g(x)| \le |f(x)| + |g(x)| \le |x| + 7 + x^2.$$

   By the reverse triangle inequality, we know that

$$|f(x) + g(x)| \ge |f(x)| - |g(x)| \ge |f(x)| - 7 - x^2$$
$$|g(x) + f(x)| \ge |g(x)| - |f(x)| \ge |g(x)| - |x|.$$

   (We *cannot* replace the $|f(x)|$ with $|x|$ in the first inequality, because knowing that $|f(x)| \le |x|$ cannot tell us anything about what $|f(x)|$ is *bigger* than. Similarly with $|g(x)|$ in the second inequality).

4. If $|f(x)| \ge 7$ and $|g(x)| \le 3$, what can we say about $|f(x) + g(x)|$ using the triangle inequalities? You should give three separate answers.

   Solution: By the triangle inequality we know that

$$|f(x) + g(x)| \le |f(x)| + |g(x)| \le |f(x)| + 3.$$

   Here we cannot do anything with the $|f(x)|$ term because we don't have an upper bound on it.

By the reverse triangle inequality, we know that

$$|f(x) + g(x)| \geq |f(x)| - |g(x)| \geq 7 - 3 = 4|g(x) + f(x)| \qquad \geq |g(x)| - |f(x)|.$$

This time we can replace $|f(x)|$ in the first inequality, since we know how *small* $f(x)$ is, not how big it is. We can't replace either $|g(x)|$ or $|f(x)|$ in the second inequality, so it's not really useful at all.

5. $\star$

   (a) Find a pair of real numbers x and y such that $|x + y| < |x| + |y|$.

   Solution: There are many correct solutions to all of these problems, but one example is $|1 + (-1)| < |1| + |-1|$.

   (b) Find a pair of real numbers x and y such that $|x + y| = |x| + |y|$.

   Solution: $|1 + 1| = |1| + |1|$.

   (c) Find a pair of real numbers x and y such that $|x + y| > x + y$.

   Solution: $|(-1) + (-1)| > (-1) + (-1)$.

6. $\star$

   (a) Find a pair of real numbers x and y such that $|x + y| > |x| - |y|$.

   Solution: $|1 + 1| > |1| - |1|$.

   (b) Find a pair of real numbers x and y such that $|x + y| = |x| - |y|$.

   Solution: $|1 + (-1)| = 1 - |-1|$.

   (c) Find a pair of real numbers x and y such that $|x + y| < x - y$.

   Solution: $|1 + (-1)| < 1 - (-1)$.