Math 114 Fall 2018
Calculus I HW 4 Solutions
Due Wednesday, October 3

1. Stewart 1.4.22
2. Stewart 1.4.24
3. Stewart 1.4.26
4. Stewart 1.6.14
5. Stewart 1.6.18
6. Stewart 1.6.20
7. Stewart 1.6.22
8. Stewart 1.6.24
9. By any means we have developed in class, compute $\lim_{x\to+\infty} x - \sqrt{x}$.

   Solution:

$$\begin{aligned}\lim_{x\to+\infty} x - \sqrt{x} &= \lim_{x\to+\infty} (x - \sqrt{x}) \cdot \frac{x+\sqrt{x}}{x+\sqrt{x}} = \lim_{x\to+\infty} \frac{x^2 - x}{x + \sqrt{x}} \\ &= \lim_{x\to+\infty} \frac{1 - 1/x}{1/x + 1/x^{3/2}}.\end{aligned}$$

   The top goes to 1 and the bottom goes to 0, so this limit is some form of infinity. Since x is approaching $+\infty$, the top and bottom are both always positive, so the limit is $+\infty$.

   Alternate solution:

$$\lim_{x\to+\infty} x - \sqrt{x} = \lim_{x\to+\infty} (x - \sqrt{x}) \cdot \frac{1/x}{1/x} = \lim_{x\to+\infty} \frac{1 - 1/\sqrt{x}}{1/x}$$

   The top goes to 1 and the bottom goes to 0, so this limit is some form of infinity. Since x is approaching $+\infty$, the top and bottom are both always positive, so the limit is $+\infty$.

10. Stewart 1.4.34
11. Stewart 1.4.36
12. ($\star$) Using the squeeze theorem, show that

$$\lim_{x\to -2} \frac{x+2}{2+\sin\left(\frac{1}{x+2}\right)} = 0.$$

Solution: We observe that

$$-1 \le \sin\left(\frac{1}{x+2}\right) \le 1$$
$$1 \le 2+\sin\left(\frac{1}{x+2}\right) \le 3$$
$$1 \ge \frac{1}{2+\sin\left(\frac{1}{x+2}\right)} \ge \frac{1}{3} \ge -1$$
$$|x+2| \ge \left|\frac{x+2}{2+\sin\left(\frac{1}{x+2}\right)}\right| \ge -|x+2|$$

Then we compute $\lim_{x\to -2} |x+2| = 0$ and $\lim_{x\to -2} -|x+2| = 0$, so by the Squeeze Theorem,

$$\lim_{x\to -2} \left|\frac{x+2}{2+\sin\left(\frac{1}{x+2}\right)}\right| = 0.$$

Thus

$$\lim_{x\to -2} \frac{x+2}{2+\sin\left(\frac{1}{x+2}\right)} = 0.$$